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Testing neoclassical consumer theory with aggregate and household data

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I. INTRODUCTION

The neoclassical theory of constrained utility maximization underlies most analysis of consumer spending across commodity groups. However, studies such as Christensen *et al.* (1975) and Deaton and Muellbauer (1980a) have rejected the neoclassical postulates of consumer theory using aggregate data, which implies that estimated price and income elasticities will not be consistent with the theory they are supposed to reflect. In this paper it is argued that the inconsistency is the result of using aggregate data.

The argument that using aggregate data leads to rejection of the theory is supported by the following test. Two data sets used to estimate the same expenditure system across eight commodities. The first is aggregate quarterly data for the US from 1959 to 1988. The second data set is multiple cross-sections from the Consumer Expenditure Survey (CES) for 25 quarters between 1980 and 1986. It is shown that neoclassical consumer theory can be rejected using the aggregate data, but not with the household data.

The tests of consistency between data and theory in consumer expenditure analysis focus on properties of demand functions. The neoclassical postulates about consumer behaviour generate four properties of demand functions – adding-up, homogeneity, symmetry, and negativity. These four properties are both necessary and sufficient results of minimizing a concave cost function. Because the four properties are sufficient, rejecting them implies that the demand functions are not consistent with neoclassical consumer theory.

Rejection of the four demand function properties can arguably lead to a rejection of neoclassical consumer theory (as opposed to just rejection of a certain set of demand functions) in the special case of the 'flexible functional form'. If the system is a flexible form, the demand functions are general enough to approximate any underlying system of preferences. The present study estimates the 'Almost Ideal Demand' (AID) system introduced by Deaton and Muellbauer (1980a, b). The AID system is a flexible form, and therefore the tests of demand function properties are tests of consumer theory as a whole, not just the specific system of equations.

In addition to testing the neoclassical theory of utility maximization, this paper also focuses on properties of systems estimated with the two types of data. Evaluations are made time-series and cross-section price and income elasticities at the means of the sample period for which the two data sets overlap, and significant differences are found in both income and price elasticities. Because the data sets were constructed so as to represent the same

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commodities, the discrepancies are attributed to having estimated the systems with different types of data.

II. TESTING CONSUMER THEORY WITH THE AID SYSTEM

Deaton and Muellbauer (1980a, b) propose the AID expenditure system, based on a flexible approximation to the consumer cost function. The budget share equation for the ith commodity in the system is,

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln (m/P) \tag{1}$$

where m is total expenditure, the p_j are individual prices, and P is a price index defined by,

$$\ln P = \alpha_0 + \sum_k \alpha_k \ln p + (1/2) \sum_k \sum_1 \gamma_{k1} \ln p_k \ln p_1 \tag{2}$$

The AID system is useful for testing consumer theory because it is a flexible form, and because demand function properties can be imposed parametrically be restrictions on the αs, βs, and γs. The system is estimated with and without the parametric restrictions, and a likelihood ratio test determines whether the restrictions can be rejected.

The first demand function property is adding-up, which requires budget shares always sum to one. The parametric restrictions which guarantee adding-up are,

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0, \quad \text{and} \quad \sum_i \beta_i = 0 \tag{3}$$

Deaton and Muellbauer point out that data sets used in expenditure analysis satisfy adding-up by construction, so that these restrictions are not testable. The case with the parameter restrictions in Equation 3 will be referred to as the 'unrestricted' system.

The alternative to be tested against the unrestricted system will have homogeneity and symmetry parametrically imposed. Homogeneity, which rules out money illusion, is satisfied in the AID system when:

$$\sum_j \gamma_{ij} = 0 \tag{4}$$

This property is testable in expenditure data. The imposition of symmetry is based on Allen–Uzuwa elasticities of substitution (AUES). AUES are compensated elasticities divided by budget shares. These elasticities have the property that equality of cross-commodity AUES implies symmetry. The cross-commodity AUES terms in the AID system are:

$$\sigma_{ij} = 1 + (\beta_i / w_i) + (1/w_i w_j)/(\gamma_{ij} - \beta_i(\alpha_j - \sum_k \gamma_{kj} \ln p_k)) \tag{5}$$

Symmetry holds, i.e. $\sigma_{ij} = \sigma_{ji}$, when $\gamma_{ij} = \gamma_{ji}$ for all i and j.

The fourth property of demand functions, negativity, cannot be imposed with *a priori* restrictions on the parameters. However, negativity can be observed (or not) by evaluating the entire AUES matrix for each data point. Negativity results if symmetry holds and the matrix of AUES terms is negative semi-definite. The diagonal elements (own-AUES) terms of the matrix are:

$$\sigma_{ii} = 1 + (\beta_i - 1)/w_i + (1/w_i^2)(\gamma_{ii} - \beta_i(\alpha - \sum_k \gamma_{ki} \ln p_k)) \tag{6}$$

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Unlike the other three properties, negativity cannot be imposed parametrically. This is a common feature of flexible form systems. The negativity condition is evaluated at every point in the sample and discussed below.

III. DATA AND ESTIMATION

The AID expenditure system described above is estimated below using two consumer spending data sets, with expenditures divided into eight categories. The first is aggregate data from the US National Income Accounts (NIA) for the period 1959 through 1988. The second data set is derived from the on-going Consumer Expenditure Survey (CES) conducted by the Bureau of Labor Statistics (BLS) for the period 1980 to 1986. The CES

Table 1 *AID system parameter estimates, time series data*

Commodity i	α_i	β_i	γ_{1i}	γ_{2i}	γ_{3i}	γ_{4i}	γ_{5i}	γ_{6i}	γ_{7i}
Unrestricted system									
Food	0.649	−0.032	0.083	−0.000	−0.036	0.039	−0.069	−0.009	−0.016
	(28.6)	(−1.9)	(9.0)	(−0.0)	(−5.1)	(6.5)	(−8.1)	(−0.5)	(−4.9)
Clothing	0.230	0.044	−0.033	−0.013	0.065	−0.029	0.002	0.068	−0.029
	(11.1)	(3.2)	(−6.7)	(−1.4)	(9.9)	(−5.4)	(0.3)	(11.5)	(−9.6)
Gas and oil	−0.127	−0.024	−0.019	−0.021	0.071	−0.008	0.013	−0.011	−0.009
	(−8.9)	(−2.5)	(−6.3)	(−8.5)	(42.9)	(−5.5)	(6.5)	(−6.5)	(10.8)
Other goods	0.188	0.016	0.046	0.028	−0.082	0.023	−0.067	−0.023	0.035
	(15.3)	(1.8)	(2.3)	(1.6)	(−6.8)	(2.2)	(−4.7)	(−1.9)	(6.4)
Hshd operation	−0.008	−0.025	−0.036	−0.006	0.009	−0.012	0.104	−0.024	−0.020
	(−0.5)	(−2.2)	(−4.4)	(−0.7)	(1.6)	(−2.4)	(14.8)	(−3.9)	(−7.5)
Transportation	−0.097	0.027	−0.048	0.000	0.007	0.006	−0.002	0.047	−0.008
	(−6.9)	(2.8)	(−7.5)	(0.0)	(1.5)	(1.5)	(−0.4)	(9.3)	(−3.3)
Personal serv	0.141	0.011	0.003	0.005	−0.040	−0.019	−0.007	0.012	0.035
	(21.5)	(2.4)	(0.4)	(0.7)	(−6.4)	(−3.4)	(−1.0)	(1.9)	(12.2)
Recreation	0.024	−0.016	−0.027	−0.040	0.062	−0.028	0.069	−0.039	−0.019
	(NA)	(NA)	(−1.1)	(−1.8)	(3.9)	(−2.1)	(3.7)	(−2.5)	(−2.7)
Restricted system									
Food	0.585	−0.065	0.096						
	(67.9)	(−18.0)	(9.5)						
Clothing	0.054	0.028	−0.013	0.068					
	(5.7)	(7.2)	(−2.7)	(12.9)					
Gas and oil	0.056	0.011	−0.019	0.004	0.044				
	(6.8)	(3.3)	(−6.5)	(1.3)	(20.1)				
Other goods	0.141	−0.020	0.018	−0.030	−0.000	−0.025			
	(21.5)	(−7.2)	(3.6)	(−7.4)	(−0.2)	(−4.3)			
Hshd operation	0.091	0.017	−0.055	−0.020	−0.002	−0.000	0.114		
	(13.3)	(6.1)	(−8.7)	(−4.9)	(−0.8)	(−0.1)	(17.0)		
Transportation	−0.009	0.032	−0.007	0.012	−0.014	−0.002	−0.012	0.035	
	(−1.5)	(12.3)	(−1.3)	(3.3)	(−7.2)	(−0.5)	(−2.7)	(6.7)	
Personal serv	0.049	−0.011	−0.026	0.006	0.002	−0.007	−0.017	0.003	0.030
	(10.6)	(−5.5)	(−8.9)	(2.0)	(1.5)	(−2.4)	(−6.1)	(1.0)	(7.7)
Recreation	0.034	0.007	0.006	−0.040	−0.029	0.034	−0.084	−0.038	−0.030
	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)

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data set is constructed to match the aggregate NIA data set, by summing over the commodity groups which comprise each of the aggregate categories.

Aggregate data is the most frequently used in estimating consumer expenditure systems. The three studies which use the testing procedure employed below (Christensen *et al.*, 1975; Berndt *et al.*, 1977; Deaton and Muellbauer, 1980a) are all based on aggregate data. All three studies reject neoclassical consumer theory.

Cross-section surveys of households have existed in the US and elsewhere for some time. However, flexible expenditure systems cannot be estimated on a single cross-section, due to the lack of price variability. Pollack and Wales (1978, 1979, 1980) have shown that if multiple cross-sections (the exact number determined by the level of commodity aggregation) are used, complete expenditure systems can be estimated. The second data set used below is

Table 2 *AID system parameter estimates, cross-section data*

Commodity i	α_i	β_i	γ_{1i}	γ_{2i}	γ_{3i}	γ_{4i}	γ_{5i}	γ_{6i}	γ_{7i}
Unrestricted system									
Food	−1.368	−0.041	−0.592	0.430	−0.241	0.801	0.244	−0.360	−0.042
	(−5.2)	(26.4)	(−3.0)	(2.2)	(−1.0)	(4.2)	(1.3)	(−2.2)	(−0.4)
Clothing	1.019	0.023	0.264	−0.260	0.169	−0.482	−0.151	0.188	−0.009
	(3.9)	(18.9)	(1.8)	(−1.8)	(1.0)	(−3.8)	(−1.2)	(1.7)	(−0.1)
Gas and oil	0.498	0.013	0.218	−0.141	0.024	−0.130	0.010	0.021	0.003
	(1.9)	(8.4)	(3.0)	(−2.4)	(0.3)	(−2.7)	(0.2)	(0.6)	(0.2)
Other goods	1.050	−0.011	−0.215	0.187	0.075	0.468	−0.183	−0.222	−0.038
	(4.0)	(10.5)	(−1.0)	(1.0)	(0.3)	(2.4)	(−1.1)	(−1.4)	(−0.4)
Hshd operation	−0.117	−0.026	−0.094	−0.319	−0.068	−0.258	0.352	0.215	0.044
	(−0.5)	(24.9)	(−0.6)	(−2.2)	(−0.4)	(−1.7)	(3.2)	(2.1)	(0.9)
Transportation	0.183	0.029	0.293	0.070	−0.042	−0.276	−0.022	−0.034	0.009
	(0.7)	(36.5)	(1.5)	(0.4)	(−0.2)	(−2.0)	(−0.2)	(−0.4)	(0.3)
Personal serv	0.053	−0.004	−0.513	0.441	0.179	−0.129	−0.407	0.337	−0.001
	(0.2)	(19.6)	(−2.3)	(2.2)	(0.8)	(−0.7)	(−2.1)	(1.9)	(−0.0)
Recreation	−0.317	0.016	1.052	−0.629	−0.188	−0.201	0.237	−0.187	0.031
	(NA)	(NA)	(5.0)	(−3.0)	(−0.9)	(−1.0)	(1.1)	(−1.0)	(0.2)
Restricted system									
Food	0.541	−0.040	0.118						
	(101.6)	(−23.4)	(0.8)						
Clothing	0.005	0.023	−0.042	−0.085					
	(1.1)	(17.9)	(−0.6)	(−1.9)					
Gas and oil	0.081	0.012	0.099	−0.042	0.092				
	(15.7)	(7.2)	(3.5)	(−2.1)	(5.0)				
Other goods	0.102	−0.011	0.276	−0.150	−0.078	0.145			
	(29.5)	(−10.0)	(3.3)	(−3.5)	(−3.6)	(1.8)			
Hshd operation	0.255	−0.026	−0.201	−0.070	−0.012	−0.260	0.185		
	(75.2)	(−24.3)	(−3.9)	(−2.5)	(−0.7)	(−6.0)	(5.0)		
Transportation	−0.011	0.030	−0.117	0.190	−0.022	0.007	0.201	−0.154	
	(−4.2)	(36.0)	(−1.7)	(5.8)	(−1.4)	(0.2)	(6.0)	(−2.7)	
Personal Serv	0.040	−0.004	−0.043	0.013	−0.002	−0.018	0.048	−0.001	−0.022
	(58.5)	(−18.1)	(−1.7)	(1.0)	(−0.3)	(−0.7)	(3.3)	(−0.1)	(−0.9)
Recreation	−0.012	0.016	−0.089	0.145	0.021	0.126	−0.433	0.156	0.022
	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)

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derived from 25 cross-sections of the CES. The 25 observations on each price are more than sufficient to estimate the parameters of an eight-commodity system.

The eight categories in the expenditure system to be estimated are food, clothing and shoes, gasoline and oil, other goods, household operation (mostly utilities), transportation services, personal services, and recreation. Three criteria were used to allocate expenditures into these eight categories. First, durable goods and housing services were dropped because of inherent intertemporal problems. Second, the remaining categories were checked for consistency between the data sets. For example, medical services in the NIA data are composed significantly of business and government health benefit transfers, which are not reflected in the budget data. Categories which suffered from this type of inconsistency were dropped. The third criterion for selecting the eight-commodity aggregation was degrees of freedom. There are 70 parameters to be estimated in the adding-up of an AID system with eight commodities; the NIA data set has only 119 observations.

Computational restrictions also forced aggregation across households in the CES data prior to estimation. There are approximately 5000 households per quarter in the CES, for a total sample size of 125 000. Experimentation indicated that samples of larger than 300 with an eight commodity system were too large for FIML estimation on an IBM 9370 computer. Therefore, the households were split into ten equal-sized income groups per quarter, with mean income and spending for each decile used as an observation, yielding 250 total observations in the sample.

The FIML estimation procedure was used, with the eighth equation in each system dropped because of singularity in the variance–covariance matrix. The assumptions which underlie the system estimation procedure are described in detail by Berndt *et al.* (1977). The parameter estimates and associated t-statistics for the unrestricted and restricted systems are shown in Tables 1 and 2.

IV. TESTS OF UNDERLYING CONSUMER THEORY

Following Christensen *et al.* (1975), the standard approach to testing demand function consistency with underlying consumer theory is the likelihood ratio test. The likelihood ratio is defined as,

$$\lambda = \left[\frac{\max_{\omega_r} L}{\max_{\omega} L} \right] \tag{7}$$

The numerator is the maximum value of the likelihood function for the system of equations given a restricted subset (ω_r) the parameter vector (ω). The denominator is the maximum value of the likelihood function when ω is not restricted. Under the null hypothesis, that the restricted parameter vector is not significantly different from the unrestricted one, the test statistic $(-2\ln\lambda)$ is distributed chi-square with degrees of freedom equal to the number of restrictions.

The values of the test statistic $(-2\ln\lambda)$ for the aggregate and cross-section data sets are 276.36 and 43.88, respectively. There are 28 restrictions; the seven coefficient sum constraints associated with homogeneity, and 21 symmetric coefficient constraints. The critical value of the chi-square distribution with 28 restrictions being tested is 48.28 at the 1% level of

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significance. The coefficient restrictions implied by neoclassical consumer theory are decidedly rejected in the aggregate data, but cannot be rejected in the cross-section data.

As noted above, the final property of demand functions, negativity, cannot be imposed and tested in the same way as homogeneity and symmetry. Negativity is satisfied at any point in a sample if the AUES matrix is negative semi-definite. Evaluation of the AUES matrix at each point in the sample indicated that negativity was not satisfied at any point by either data set. It should be stressed that there is no statistical significance to be attached to this result. The estimated AUES terms are themselves surrounded by (complicated) confidence intervals; it would be necessary to show that negativity does not occur at any values for elasticities within a given probability interval in order to reject it with some measure of certainty.

The above results for the aggregate data set are consistent with those found by other authors. Deaton and Muellbauer (1980a) are able to reject the neoclassical postulates, and find failure of negativity in their data. The authors suggest several possible reasons for the failure of the theory, including inflexible spending patterns in the short run, changing expectations about future prices, and non-separability of the group of commodities being studied. Deaton and Muellbauer suggests controlling for these as important lines of future research.

The fact that the theory is not rejected by the cross-section data sheds new light on the theory's inability to explain the aggregate data. Because the cross-section data are also quarterly, the hypotheses stated above should also be applicable. Therefore, it seems that rejection of the theory in the aggregate data is caused by either structural shifts in tastes over the long time period being studied, or reliance on the average consumer being representative. In either case, using multiple cross-sections alleviates the problem.

V. COMPARISON OF SYSTEM PROPERTIES

In addition to testing for consistency of expenditure systems with theory using the two data sets, it is interesting to look at the estimated price and income elasticities. As in the tests of theory, the fact that the two data sets were constructed to match suggests that any differences are attributed to using the different types of data. The estimated elasticities, evaluated at the means of the samples for the period in which they overlap, are shown in Table 3.

Table 3. *AID system estimated elasticities (evaluated at sample means, 1980 1 to 1986.1)*

	Own-price elasticities		Income elasticities	
	Aggregate data	Cross-section data	Aggregate data	Cross-section data
Food	−0.32	−0.29	0.85	0.90
Clothing	−0.30	−2.05	1.23	1.32
Gas and oil	−0 37	−0.07	1.13	1.11
Other goods	−1 16	1.05	0.79	0.85
Hshd operation	−0.01	0.24	1.13	0.85
Transportation	−0.39	−2.87	1.47	1 38
Personal serv	0.30	−1.75	0 55	0 86
Recreation	−0 96	−5.49	1.14	1 43

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There are striking differences in estimated price and income elasticities for some of the eight commodities in the system. In terms of price elasticities, the first thing to note is the observation of positive (compensated) price elasticities for two of the goods in the cross-section data, and one good in the aggregate data, consistent with the observation that negativity fails in both data sets. It is also clear that price elasticities vary in direction as well as magnitude in the two data sets.

All of the commodities in the aggregate data are price inelastic except 'other goods'. This is consistent with the findings of Deaton and Muellbauer (1980a), who found inelastic price effects to be the rule. However, the comparative price elasticities across commodities are not consistent with earlier results. For example, 'gas and oil' is more price elastic than 'food', which in turn is more price elastic than 'clothing'. This reverses the ordering in Deaton and Muellbauer's study, and the ordering observed in the cross-section data.

Even though the cross-section price elasticities are ordered in what appears to be a reasonable way, the magnitudes are extreme. Three price elasticities are above two in absolute value. There is no reason, however, to reject these estimates out of hand in favour of the aggregate price elasticities. In addition to the problem of how the price elasticities in the aggregate data are ordered across commodities, it appears that some of the aggregate price effects are too low in magnitude. For example, 'transportation' includes some goods and services which should be price inelastic, such as auto repair, but also some highly elastic components such as airline travel. It is not clear that the aggregate price elasticity of -0.39 is more reasonable than the cross-section estimate of -2.87.

The two data sets are more in agreement about income elasticities. The only difference in terms of elastic/non-elastic income response is in the 'household operation' category, which is mostly fuel, electric, and other utilities. The aggregate data indicate that this commodity group is income elastic, whereas the cross-section data indicate it is inelastic which seems more reasonable. Beyond this, the ordering of most income elastic to least income elastic across the two data sets is consistent, and similar to that found elsewhere. The only startling difference between these income elasticities and those estimated by Deaton and Muellbauer (1980a) is the 'food' category; the income elasticities in both data sets used here are much higher. The resolution of this could be that the data used here includes food at restaurants, which is more income elastic.

VI. CONCLUSIONS

The implementation of neoclassical consumer theory has been shown to be very sensitive to the type of data used to estimate the system. Both a time-series aggregate data set and multiple cross-section data set for the same commodities were estimated, yielding different results about consistency with the neoclassical theory and estimated price and income elasticities. The aggregate data are inconsistent with the theory, whereas the cross-section data is not.

There are many applications in macroeconomic modelling and microeconomic welfare analysis which are affected by this result. In macroeconomic modelling, estimated price and income elasticities are extremely important for general equilibrium properties (for example, Almon, 1979). Estimated elasticities also form the basis for welfare analysis in evaluation of transfer and/or subsidy programmes at the microeconomic level.

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These applications are suspect if the estimated systems are found not to be consistent with theory upon which they are based. The results of this paper suggest consistency can be found in multiple cross-sections, but not in aggregate data. Given that estimated system price and income elasticities also vary greatly across the two types of data, the decision about which to use becomes crucial to the application. Development of complete cross-section data sets and resolution of the discrepancies between the two types of data are important lines of future research.

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