Econometrica, Vol. 60, No. 3 (May, 1992), 597–625

BARGAINING AND THE RIGHT TO REMAIN SILENT

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This paper analyzes a class of alternating-offer bargaining games with one-sided incomplete information for the case of "no gap." If sequential equilibria are required to satisfy the additional restrictions of stationarity, monotonicity, pure strategies, and no free screening, we establish the Silence Theorem: When the time interval between successive periods is made sufficiently short, the informed party never makes any serious offers in the play of alternating-offer bargaining games. A class of parametric examples suggests that the time interval required to assure silence is not especially brief.

As a byproduct of the analysis, we also prove (under the same set of assumptions) a uniform version of the Coase Conjecture: When the time interval between successive periods is made sufficiently short, the initial serious offer by either party in an alternating-offer bargaining game must be less than ε times the highest possible buyer valuation, for an entire family of distribution functions.

KEYWORDS: Noncooperative bargaining, alternating offers, stationarity, Coase Conjecture, incomplete information.

1. INTRODUCTION

THE NEGOTIATION PROCESS TRANSMITS INFORMATION in at least two ways. First, any time that an informed party responds (positively or negatively) to an existing offer on the bargaining table, he may reveal some of his private information to his partners in the negotiations. Second, whenever that party places his own new counteroffer on the table (or refrains from doing so), the form of the proposal potentially conveys some information. Together, these two vehicles for information transmission may result in the rapid disclosure of the informed party's information.

Consider a bilateral bargaining situation where one of the parties possesses private information which the other party wishes to learn. It is reasonable to think that the first channel ("passive revelation") can be more readily exploited to expose the informed agent's information than can the second channel ("active revelation"). The uninformed agent obtains information via passive revelation by making an offer, which the informed agent finds either attractive

[1] This research was supported in part by National Science Foundation Grant SES-86-19012. Financial assistance was also provided, for Lawrence Ausubel, by the Lynde and Harry Bradley Foundation, and for Raymond Deneckere, by the Kellogg School of Management's Beatrice/Esmark Research Chair. The idea for this paper arose during a visit of both authors to the Institute for Mathematical Studies in the Social Sciences at Stanford University in the Summer of 1986. We would like to thank the IMSSS for its hospitality, and Faruk Gul and Robert Wilson for an especially valuable conversation we had at that time. We also thank Elhanan Ben-Porath, Martin Hellwig, Jean-François Mertens, Daniel Vincent, and two anonymous referees for helpful comments, as well as seminar participants at the Midwest Mathematical Economics Meetings, the World Congress of the Econometric Society, Columbia University, Katholieke Universiteit Leuven, New York University, Northwestern University, Stanford University, State University of New York at Stony Brook, University of California at Berkeley, University of California at San Diego, University of Illinois at Urbana-Champaign, University of Iowa, and University of North Carolina. Any errors in the paper are wholly our responsibility.

or unattractive, depending on his private information, and merely waiting for the informed agent's response. In contrast, active revelation relies on the informed party's willingness to voluntarily choose to frame a proposal which reveals his information. The uninformed party can utilize the first device to force the informed party to disclose; however, the informed party has the option of refraining from making counteroffers, and thus can avoid the second means of information transmission.

In this article, we formally derive a result of this type. Consider the (k, l)-alternating-offer bargaining game[2] of one-sided incomplete information. Restrict attention to the set of sequential equilibria which satisfy the additional restrictions of stationarity, monotonicity, pure strategies, and no free screening.[3] Our main result then is the Silence Theorem: there exists a sufficiently short (but still positive) time interval between successive offers such that *the informed party never makes any serious counteroffers* in any of these equilibria.[4] All information revelation then occurs only through passive responses by the informed party to offers of the uninformed party.

Our result thus provides a justification for studying the bargaining game of one-sided incomplete information in which only the uninformed party is permitted to make offers. This game, while extensively and successfully studied in earlier papers,[5] has also been criticized for artificially restricting the actions of the informed party.[6] In contrast, our current result establishes that, for an interesting class of equilibria, the outcome of an alternating-offer game is *as if* the extensive form permitted offers only by the uninformed party.[7] Exogenously, both traders are permitted to make offers; endogenously, equilibrium counteroffers by the informed party degenerate to null moves.

There is a simple intuition for the Silence Theorem. Our restrictions on sequential equilibrium mandate that, at each of his moves in the game, the

[2] We introduced this terminology in Ausubel and Deneckere (1989b): k offers by the uninformed agent are followed by l counteroffers by the informed agent, whereupon the game repeats until agreement is reached. The $(1,1)$ extensive form is the standard alternating-offer game introduced by Rubinstein (1982).

[3] The assumptions of stationarity, monotonicity, pure strategies, and no free screening were introduced by Gul and Sonnenschein (1988). The structure in which informed agents' valuations are partitioned into exactly two subintervals at each informed agent move (which is implicit in the assumptions of pure strategies and no free screening) was introduced earlier by Grossman and Perry (1986); it is necessitated by their notion of perfect sequential equilibrium.

[4] More precisely, the Silence Theorem holds both along the equilibrium path and after any history in which the informed agent has not previously deviated from his equilibrium strategy.

[5] Papers on the bargaining game with one-sided incomplete information where the uninformed party makes all the offers, and the related problem of durable goods monopoly, include: Bulow (1982), Stokey (1981), Fudenberg and Tirole (1983), Sobel and Takahashi (1983), Fudenberg, Levine, and Tirole (1985), Gul, Sonnenschein, and Wilson (1986), and Ausubel and Deneckere (1989a, b).

[6] For example, see footnote 2 of Grossman and Perry (1986).

[7] More precisely, we compare: the set of sequential equilibrium outcomes (that satisfy stationarity, monotonicity, pure strategies, and no free screening) of the alternating-offer bargaining game with time interval z between periods; and the set of stationary sequential equilibrium outcomes of the seller-offer bargaining game with time interval $2z$ between periods. We show that, provided z is sufficiently small, the two sets exactly coincide.

informed agent partitions the interval of remaining possible valuations into two subintervals (one possibly degenerate). In particular, at times when it is the informed agent's turn to make an offer, the remaining valuations partition into a high subinterval (who *speak* by making a serious offer) and a low subinterval (who effectively *remain silent* by making a nonserious offer). Now consider the dilemma from the viewpoint of the informed party: you have the options of speaking or remaining silent. Choosing to speak reveals a high valuation, which is information that your bargaining partner can exploit in the ensuing negotiations. Remaining silent indicates a low valuation, at the cost of delaying agreement until the next round of offers. As the time between offers shrinks toward zero, the terms of trade for a low-valuation type become increasingly favorable: à la the Coase Conjecture, the seller's price converges to zero. Meanwhile, the cost of delay becomes arbitrarily low, whereas the revelation of a high valuation becomes increasingly injurious (since the adverse information can be exploited more quickly). Thus, silence becomes increasingly attractive relative to speaking and, for sufficiently short time intervals, delay is preferable to revealing the damaging information for *all* types of the informed party. In other words, you recognize that "anything you say can and will be used against you." Therefore, regardless of valuation, you decline to speak, since "you have the right to remain silent."[8]

It is important to emphasize that the above logic, which may require all types of the informed agent to pool together when *making* offers (therefore all making a nonserious offer[9]), does not also require all types to pool in the course of *accepting* offers. While accepting an offer may equally reveal that the informed party's valuation is high, there is a fundamental asymmetry between offer and acceptance in a bargaining game: an acceptance has the effect of immediately ending the game. There is no subsequent play in which the uninformed agent can exploit the favorable information conferred by an acceptance (and a rejection conveys only unfavorable information).

The existing article most closely related to the present paper, and on which we significantly rely, is that of Gul and Sonnenschein (1988). Gul and Sonnenschein examined the standard (1, 1)-alternating-offer game under one-sided incomplete information, for the case of a "gap" between the uninformed party's valuation and the (lowest possible) informed party's valuation. They formulated the four restrictions on sequential equilibrium and demonstrated

[8] The two phrases quoted in this paragraph were taken from the standard "rights card" used by the San Francisco Police Department in the aftermath of the *Miranda* decision. See *American Jurisprudence Proof of Facts*, Bancroft-Whitney Co., San Francisco, 1967, Vol. 19, p. 80, and *Miranda v. Arizona*, 384 U.S. 436 (1966).

[9] For the basic distributional assumption under which the Silence Theorem holds—namely, that there is "no gap" between the uninformed party's valuation and the (lowest possible) informed party's valuation—there cannot exist a fully-pooling *serious* buyer offer. This follows from the fact that any serious offer must be individually rational for all types who make it, and so any fully-pooling serious offer would need to be less than or equal to the valuations of all types of the informed party. With "no gap," this in turn is less than or equal to the uninformed party's valuation. However, in any equilibrium, the uninformed party rejects all such offers.

that these imply the "no delay" result: for any $\varepsilon > 0$, there exists a sufficiently short (but still positive) time interval between offers such that the probability of trade within time ε exceeds $1 - \varepsilon$. Our departure from Gul and Sonnenschein is two-fold. First, we prove a *uniform version* of the Coase Conjecture[10] for *(k, l)-alternating-offer games* in the case of *no gap*.[11] Second, we use the uniform Coase Conjecture merely as an auxiliary result in proving our main theorem, that the informed party never speaks.

Grossman and Perry (1986) examine alternating-offer bargaining in the case of a gap and prove that there exists at most one "perfect sequential equilibrium." Ausubel and Deneckere (1989b) characterize the entire set of sequential equilibria for the (k, l)-alternating-offer game in the case of no gap.[12] Rubinstein (1985) considers alternating-offer bargaining where the uncertainty concerns the rate of time preference of the informed party. In a model with two types, he shows that there is a continuum of sequential equilibria but generally a unique "bargaining sequential equilibrium." Admati and Perry (1987) examine a different alternating-offer, extensive-form game which circumvents the no delay result.

The structure of our article is as follows. In the next section, we describe the model and the equilibrium concept. In Section 3, we prove the Silence Theorem. In Section 4, we discuss the relationship with finite-horizon bargaining games and with the signaling literature. We conclude in Section 5 by showing that silence is mandatory even when the time interval between offers is relatively long (and so the Coase Conjecture has little force).

2. THE MODEL

Consider a situation where two parties are bargaining over the price at which a single item is to be sold. The seller's valuation for the object is common knowledge, for convenience normalized to equal zero. However, the buyer's valuation is private information, drawn from the (commonly-known) distribution function $F(\cdot)$. Let $\underline{b}$ and $\overline{b}$, respectively, denote the lower and upper ends of

[10] The no delay result is closely connected to the Coase Conjecture for durable goods monopoly and bargaining where the uninformed seller makes all the offers. The Conjecture states that, for any $\varepsilon > 0$, there exists a sufficiently short (but still positive) time interval between offers such that the initial offer is always within ε of the lowest buyer valuation.

Coase (1972) introduced the intuition for the Conjecture. Gul, Sonnenschein, and Wilson (1986) proved the Coase Conjecture to hold for the case of "the gap," and to be true for "no gap" under an assumption of stationarity. In Ausubel and Deneckere (1989a), we showed the Conjecture to be false for "no gap" without this additional assumption.

In the alternating-offer game, a Coase Conjecture type result (such as Theorem 3.2 below) implies no delay—since the initial offer is very low, buyer acceptance occurs very quickly. For the case of "no gap," it can conversely be shown that a no delay result would imply the Coase Conjecture.

[11] The no delay result of Gul and Sonnenschein was only proven for the case of *a gap*. In addition, a *uniform* version of the Coase Conjecture (i.e., that prices are uniformly low, relative to the state, as the game evolves) is required for proving the Silence Theorem.

[12] We also proved that, for both the gap and no gap cases, the game where only the informed party makes offers has a unique sequential equilibrium.

the support of $F(\cdot)$, i.e., $\underline{b} = \inf\{b\colon F(b) > 0\}$ and $\bar{b} = \sup\{b\colon F(b) < 1\}$. Our main substantive restriction on $F(\cdot)$ will be the assumption that there is *no gap* between the seller's and the lowest buyer's valuation, i.e., $\underline{b} \leqslant 0$.[13] (In contrast, Gul and Sonnenschein (1988) restrict attention to the case of a *gap*, i.e., $\underline{b} > 0$; we briefly discuss the case of a gap in footnote 30, below.)

In order to easily accommodate distributions containing mass points, it is convenient to adopt the following transformation for the remainder of this article.[14] For every $q \in I = [0,1]$, define $f(q) = \inf\{b \in [\underline{b}, \bar{b}]\colon F(b) \geqslant 1 - q\}$. Then, without any loss of generality, we may assume that q is uniformly distributed on the unit interval: q may be viewed as the buyer's type; each *type* q has *valuation* $f(q)$. Note that, by definition, $f(\cdot)$ is a left-continuous and weakly-decreasing function. As a simple normalization, we also assume $f(q) > 0$ for $q \in [0,1)$, $f(0) = 1$, and $f(1) = 0$.[15] The basic purpose of replacing the distribution function $F(\cdot)$ with a valuation function $f(\cdot)$ is so that any truncation would be fully described by the endpoints of the truncated support.[16] Nonetheless, our transformed notation admits completely general distributions, as $f(\cdot)$ is permitted to exhibit (right) discontinuities and flat regions.

The seller and buyer are both impatient; in fact, we assume a common discount rate of r. Thus, if trade occurs at a price p at time t, the seller derives a net surplus of pe^{-rt}, and the buyer (of type q) earns $[f(q) - p]e^{-rt}$.

Players alternate in making offers at discrete moments in time, spaced z apart. Thus, the discount factor between successive periods is $\delta \equiv e^{-rz}$. The seller proposes in even periods (by convention, the initial period is taken to be zero) and the buyer proposes in odd periods. Immediately after an offer has been made, the other party can either accept or reject the offer. Acceptance terminates the game; rejection yields the opportunity to make a counteroffer in the next period. Let h_n denote an n-period history of prices and rejections, and let H_n denote the set of all h_n. Let h'_n denote h_n followed by a price offer in period n, and let H'_n denote the set of all h'_n. The strategy of the seller is a sequence of functions $\sigma^s = \{\sigma_n^s\}_{n=0}^{\infty}$, where $\sigma_n^s\colon H_n \to \mathbb{R}$ for n even and $\sigma_n^s\colon H'_n \to \{Y, N\}$ for n odd. Similarly, the strategy for the buyer is a sequence of functions $\sigma^b = \{\sigma_n^b\}_{n=0}^{\infty}$, where $\sigma_n^b\colon H'_n \times I \to \{Y, N\}$ for n even and $\sigma_n^b\colon H_n \times I$

[13] If $F(\cdot)$ is not strictly monotone within its support, the assumption of "no gap" should be more precisely stated as: for every $\varepsilon > 0$, there exists $b \in (0, \varepsilon)$ contained in the support of $F(\cdot)$. The transformation of the next paragraph will also require that $\bar{b} < \infty$.

[14] The function $f(\cdot)$ is precisely the (inverse) demand function of the analogous durable goods monopoly problem—see Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989a).

[15] Any buyer types with valuations less than or equal to the seller's are not effective players in the game and hence are deleted.

[16] A truncated distribution function $F(\cdot)$ is *not* fully specified by the endpoints of the support. Indeed, suppose that $F(\cdot)$ has a mass point at $\hat{b} \in (\underline{b}, \bar{b})$. Then the statement that "$F(\cdot)$ has been truncated to the subinterval $[\underline{b}, \hat{b}]$" does not unambiguously describe the posterior distribution. We need to additionally know *what portion of the mass point* remains and what portion is gone. In contrast, if we had been told that the associated valuation function $f(\cdot)$ was truncated to a specified subinterval, then the posterior would have been completely specified (up to sets of measure zero).

If $F(\cdot)$ has no mass points, then we do not require the transformed notation.

$\to \mathbb{R}$ for n odd. We assume that the buyer's strategy is measurable in the second argument (his type). A strategy profile is denoted by $\bar{\sigma} = \{\sigma^s, \sigma^b\}$.

Let W denote the set of probability distributions on I and let $Z \subset W$ denote the set of uniform distributions on intervals $[a, b]$, where $0 \leqslant a \leqslant b \leqslant 1$. Distributions in Z will be denoted by the endpoints of their supports (e.g., $(a, b) \in Z$). Seller beliefs are defined for each history of the game by functions g_n: $H_n \to W$ and g'_n: $H'_n \to W$. Specifically, g_n denotes the seller's beliefs at the start of period n, and g'_n denotes her beliefs following the offer of the period n. We require that these beliefs do not change after the seller's own move, that is, $g'_{n-1} = g_n = g'_n$ for n even. Finally, let $\bar{g}_n \equiv \{g_n, g'_n\}$ and let $\bar{g} \equiv \{\bar{g}_n\}_{n=1}^{\infty}$.

We also require that, for every odd-numbered period n with beliefs $g_n = g_n(h_n)$, the updated beliefs g'_n (after the buyer names a price) be consistent with Bayes' rule as applied to the strategy σ_n^b and the beliefs g_n, even if the beginning-of-period history h_n is off the equilibrium path. For every even-numbered period, we analogously require Bayesian updating (after the buyer rejects an offer), again even if h'_n is off the equilibrium path.[17] Furthermore, strategies must be sequentially rational in the sense that, at every information set, a player's strategy maximizes his expected payoff, given his beliefs and his opponent's strategy. Every pair $[\bar{\sigma}, \bar{g}]$ of strategies and beliefs with the above properties will be referred to as a "sequential equilibrium."[18]

It is well known that in the seller-offer game, where the buyer has no opportunities to make a counteroffer, the seller successively skims through the buyer's possible valuations. A somewhat analogous proposition remains true in even periods of the alternating-offer game.[19]

LEMMA 2.1: *For any sequential equilibrium and any even number n there exists a function* Q: $H'_n \to I$ *such that for all* $h'_n \in H'_n$, $\sigma_n^b(h'_n, q) = Y$ *if and only if* $q \leqslant Q(h'_n)$.

[17] For finite games, this property is implied by the consistency requirement in the definition of sequential equilibrium (Kreps and Wilson (1982, p. 872)). We make essential use of this property in the first paragraph of the proof of Theorem 3.3.

[18] For finite games, Fudenberg and Tirole (1988) have called this concept a "perfect Bayesian equilibrium." The solution concept is weaker than Kreps and Wilson's (1982) sequential equilibrium, since it does not constrain beliefs at period $(n+1)$ information sets which have zero probability of being reached given the period n beliefs and equilibrium strategies. (Sequential equilibrium does constrain these beliefs; for an illuminating example, see Fudenberg and Tirole (1988, Section V).) Perfect Bayesian equilibrium also permits a type with zero prior probability to be assigned a positive posterior probability following a zero-probability history.

Neither terminology is ideal in our context, as the bargaining game is an infinite game.

[19] The proof of this lemma is standard; see, e.g., Fudenberg, Levine, and Tirole (1985, Lemma 1), and Ausubel and Deneckere (1989a, Lemma 2.1). For expositional ease, the statement of the lemma assumes that the buyer uses pure acceptance strategies and that buyer types with valuations equal to $f(Q(h'_n))$ choose monotonically between acceptance and rejection. Without this restriction to pure strategies, the reader should observe that any buyer type with valuation exactly equal to $f(Q(h'_n))$ may respond "Y", "N", or randomize.

While buyer accept/reject moves thus lead to a truncation of the seller's beliefs concerning the buyer's type, the same need not be true about buyer offers. Following Gul and Sonnenschein (1988) and Grossman and Perry (1986), we will henceforth make two assumptions which guarantee that buyer offers also truncate the seller's beliefs.

A.1 (PURE STRATEGIES): *After histories with no prior buyer deviations, the seller's offer and acceptance behavior is deterministic.*[20]

A.2 (NO FREE SCREENING): *For all odd n, and for all $h_n \in H_n$, let $\psi(h_n) = \{p: \sigma_n^b(h_n, q) = p$ for some $q \in I\}$. Then if $p, p' \in \psi(h_n)$ and $g_n'(h_n, p) \neq g_n'(h_n, p')$, either $\sigma_n^s(h_n, p) = Y$ or $\sigma_n^s(h_n, p') = Y$.*

A.1 guarantees that at any stage of the game both the seller and the buyer make at most one *serious* offer (an offer which has positive probability of acceptance). Indeed, for the seller, this assumption directly guarantees that no randomization occurs, and hence that there is at most one serious offer. The fact that the seller never accepts offers with a probability in $(0, 1)$ makes it suboptimal for the buyer to make more than one serious offer in any given period (the lowest acceptable offer always dominates). A.2 rules out cheap talk, that is, non-payoff-relevant moves which reveal information. More precisely, the seller is required to form the same update following different *nonserious* offers (offers which have zero probability of acceptance). Without loss of generality, we will henceforth assume that there is a unique nonserious offer in each period. The two assumptions taken together imply that the equilibrium paths of our equilibria display a simple and intuitive structure.

LEMMA 2.2: *For every sequential equilibrium $(\bar{\sigma}, \bar{g})$ satisfying* A.1–A.2, *there exists a unique nondecreasing sequence $q_0, q_1, q_2, \ldots$ called the states generated by $\bar{\sigma}$, and for each i there exists a unique $h_i \in H_i$ (and if i is even a unique $h_i' \in H_i'$) that occur with positive probability under $\bar{\sigma}$, such that:*

(i) *if i is even, then $\sigma_i^b(h_i', q) = Y$ if $q \in (q_i, q_{i+1}]$ and $\sigma_i^b(h_i', q) = N$ if $q \in (q_{i+1}, 1]$;*

(ii) *if i is odd, then $\sigma_i^b(h_i, q) = p_i$ if $q \in (q_i, q_{i+1}]$ and p_i is the unique serious offer at i, and $\sigma_i^b(h_i, q) = \bar{p}_i$ if $q \in (q_{i+1}, 1]$ and $\bar{p}_i$ is the unique nonserious offer at i.*

Furthermore, if i is even, then $g_i(h_i) = g_i'(h_i') = (q_i, 1)$. If i is odd, then $g_i(h_i) = (q_i, 1)$; in addition, $g_i'(h_i, p_i) = (q_i, q_{i+1})$ and $g_i'(h_i, \bar{p}_i) = (q_{i+1}, 1)$.

[20] With the exception that the seller is permitted to use a mixed strategy in naming her first offer following a deviation from the seller's equilibrium strategy. Such mixing may be needed to permit existence of equilibrium, once the subsequent stationarity assumptions are introduced. See Fudenberg, Levine, and Tirole (1985, p. 80).

PROOF: See Gul and Sonnenschein (1988).

Equilibria satisfying A.1–A.2 may have players' strategies depending on the entire history of offers and counteroffers. In the bargaining context, many authors (Gul, Sonnenschein, and Wilson (1986), Gul and Sonnenschein (1988), Cho (1990)) have advocated restricting attention to equilibria in which the buyer's strategy is Markovian, i.e., where the buyer's behavior is allowed to depend on the previous history only insofar as it is reflected in the current state. Stationary or Markovian equilibria have also been widely promoted in other economic contexts, and generally two classes of defense are offered. First, Markovian equilibria have been argued to be attractive on the grounds of simplicity: use of stationary strategies may place fewer computational and informational demands on players and the resulting equilibria may thus be viewed as focal.[21] Second, Markovian restrictions may be viewed as natural in that they require players to pay attention only to the portion of history which is payoff-relevant.[22] While we do not find these arguments entirely persuasive, we believe it is interesting to explore the consequences of stationarity.[23,24] We will therefore follow Gul and Sonnenschein (1988) in making the additional assumptions:

A.3 (STATIONARITY OF THE BUYER'S OFFER BEHAVIOR): *For every n and m odd,* $q \in I$, *and every* $h_n \in H_n$ *and* $h_m \in H_m$, $g_n(h_n) = g_m(h_m)$ *implies* $\sigma_n^b(h_n, q) = \sigma_m^b(h_m, q)$.

[21] For example, Maskin and Tirole (1988, p. 553) write: "We have several reasons for restricting our attention to Markov strategies. Their most obvious appeal is their simplicity. Firms' strategies depend on as little as possible while still being consistent with rationality." Myerson (1991, p. 112) writes: "In repeated games, simplicity or stationarity of the strategies in an equilibrium may make that equilibrium more focal, other things being equal."

Perhaps the most alluring feature of stationary or Markovian equilibria is the apparent ease with which agents can carry out their equilibrium strategies. In particular, agents do not find it necessary to remember or analyze a potentially long history of actions; the current state is a sufficient statistic.

[22] Stationarity is an implication of Harsanyi and Selten's (1988) principle of subgame-consistency, which requires that behavior in a subgame depend only on the structure of that subgame (and should be independent of any larger game in which the subgame is embedded). A related condition is invoked in Kalai and Samet (1985). Similarly, Maskin and Tirole (1988, p. 552) write: "We do not accept any perfect equilibrium, however, but just those whose strategies depend only on the "payoff-relevant" history. Specifically, at time $t = 2k$, the only aspect of history that has any "direct" bearing on current or future payoffs is the value of a_{2k-1}^1, for only this variable, among all those before time $2k$, enters any instantaneous profit function from time $2k$ on."

[23] For example, some of our comments in Ausubel and Deneckere (1989a, b) are implicitly quite critical of stationarity.

[24] A third defense which has been advanced for stationarity restrictions is that they, in some sense, confine attention to equilibria of the infinite-horizon game which are limits of equilibria of finite-horizon versions of the same game. This also makes the computation of a stationary equilibrium, via backward recursion, relatively easy.

The correspondence between limits of finite-horizon equilibria and stationary equilibria holds true in many complete-information environments. In addition, in the bargaining game of one-sided incomplete information in which the uninformed party makes all the offers, the limit of equilibria of finite-horizon versions is a stationary equilibrium of the limit game (although there also exist others; see Gul, Sonnenschein, and Wilson (1986, Section 4)). However, the correspondence is only "half-true" for the alternating-offer game: it holds for only one of the two natural finite-horizon versions. We will explore this in detail in Section 4a.

A.4 (Monotonicity of the Buyer's Acceptance Behavior): *For every n and m even, for every $h_n \in H_n$, $h_m \in H_m$, and $q_m \leqslant q_n$ such that $g_n(h_n) = (q_n, 1)$ and $g_m(h_m) = (q_m, 1)$, and for every $p \in \mathbb{R}$, there exists $p^* \geqslant p$ such that $\sigma_m^b((h_m, p^*), q) = \sigma_n^b((h_n, p), q)$ for every $q \in I$.*[25]

Assumption A.4 is actually a hybrid assumption, requiring not only a certain type of monotonicity in the buyer's acceptance behavior as a function of the current state, but also stationarity of his acceptance behavior. To understand the precise meaning of A.4, let us define, for each n even and each $h_n \in H_n$ with $g_n(h_n) = (q, 1)$, the acceptance function $P(h_n, x) = \sup\{p\colon Q(h_n, p) \geqslant x\}$. Thus, $P(h_n, x)$ is the highest price a buyer of type x will accept after history h_n.

It is straightforward to verify that if m is even, and if $h_m \in H_m$ is such that $g_m(h_m) = (q, 1) = g_n(h_n)$, then $P(h_n, \cdot) = P(h_m, \cdot)$. Thus, A.4 implies that the buyer's acceptance is only a function of the current state. Furthermore, it is straightforward to verify that if m is even and, instead, $g_m(h_m) = (q', 1)$ for some $q' < q$, then $P(h_m, \cdot) \geqslant P(h_n, \cdot)$. Thus, the buyer's acceptance behavior is monotone in the sense that the presence of additional high-valuation buyer types (those in the interval $(q', q]$) does not lead a particular buyer type to lower his acceptance price.

One final remark: the reader should observe that, in any stationary sequential equilibrium and in any "cycle" of offer and counteroffer, there must be a positive probability of trade. Indeed, suppose that there were two consecutive periods in which only nonserious offers were made. By stationarity, the buyer would continue to make nonserious offers. The seller then must eventually make a serious offer, since there always exists a positive price which has a positive probability of acceptance. Note that she could have accelerated this offer by two periods, and stationarity would have assured her the same continuation profits. This contradicts the optimality of the seller's strategy.

3. THE SILENCE THEOREM

In this section, we establish the main result of the paper. It is useful to begin with two auxiliary results. The first is a lemma which is closely related to Lemma 3.1(iii) of Grossman and Perry (1986).[26] Our lemma in effect states that, once the seller has narrowed her beliefs on the buyer's type to a subset of the interval $[0, \bar{q}]$, the most "pessimistic" equilibrium belief which can follow is that the

[25] A.3 and A.4 only need to be assumed for histories in which the buyer has not previously deviated.

[26] Our Lemma 3.1 treats all sequential equilibria, whereas Grossman and Perry's Lemma 3.1 concerns only sequential equilibria which satisfy the so-called "support restriction." According to this restriction, a revision in beliefs should not increase the support of the distribution representing the uninformed party's beliefs (Grossman and Perry (1986, footnote 5)). While such a restriction may seem natural (in particular, it must hold along the equilibrium path), it is actually very strong, and prevents an equilibrium from existing in some contexts (see Madrigal, Tan, and Werlang (1987)).

buyer's type equals $\bar{q}$ with probability one. Consequently, in equilibrium, the seller never settles for less than the equilibrium offers of the Rubinstein (1982) game between a seller of valuation zero and a buyer of valuation $f(\bar{q})$. We have the following lemma.

LEMMA 3.1: *For any valuation function $f(\cdot)$, consider any sequential equilibrium of the alternating-offer bargaining game. Suppose, after any history in which the buyer has not previously deviated, the seller maintains beliefs that the buyer's type is at most $\bar{q}$ (and so the buyer's valuation is at least $f(\bar{q})$). Then the seller will reject any counteroffer less than $(\delta/(1+\delta))f(\bar{q})$ and will not offer any price less than $(1/(1+\delta))f(\bar{q})$.*

PROOF: Define $\bar{p}$ to be the infimum over all prices which the seller accepts or offers in any sequential equilibrium, after any history in which the buyer has not previously deviated and in which the seller maintains beliefs that the buyer's type is at most $\bar{q}$. Since acceptance is individually rational, the seller never accepts an offer (strictly) less than zero. It then follows from the reasoning in Fudenberg, Levine, and Tirole (1985, Lemma 2) that the seller never offers less than zero. This establishes that $\bar{p} \geqslant 0$, a bound which we will now tighten.

In any even-numbered period, after any history as above, the surplus to buyer q in the continuation game (following rejection) is bounded above by $\delta[f(q) - \bar{p}]$. Knowing this, all buyer types $q \in [0, \bar{q}]$ accept offers p satisfying $f(\bar{q}) - p > \delta[f(\bar{q}) - \bar{p}]$ or, equivalently, $p < (1-\delta)f(\bar{q}) + \delta\bar{p}$. Consequently, any seller offer p satisfies $p \geqslant (1-\delta)f(\bar{q}) + \delta\bar{p}$.

Suppose that $\bar{p} < (\delta/(1+\delta))f(\bar{q})$. Then for any $\varepsilon > 0$, there exists p satisfying $\bar{p} \leqslant p < \bar{p} + \varepsilon$ such that after some history in which the buyer has not previously deviated and in which the seller maintains beliefs that the buyer's type is at most $\bar{q}$, the seller accepts or offers the price p. Consider

$$\varepsilon = \tfrac{1}{2}(1-\delta^2)\left[\left(\frac{\delta}{1+\delta}\right)f(\bar{q}) - \bar{p}\right].$$

Observe that $p < (1-\delta)f(\bar{q}) + \delta\bar{p}$; by the previous paragraph, p must be a buyer offer. The seller has the option of rejecting p and counteroffering $p' = (1-\delta)f(\bar{q}) + \delta\bar{p} - \varepsilon/\delta$. Again by the previous paragraph, p' is accepted with probability one. Observe that $\delta p' = \bar{p} + \varepsilon > p$, so that the deviation is profitable for the seller. We conclude that $\bar{p} \geqslant (\delta/(1+\delta))f(\bar{q})$. Finally, since any seller offer p satisfies $p \geqslant (1-\delta)f(\bar{q}) + \delta\bar{p}$, we also have $p \geqslant (1/(1+\delta))f(\bar{q})$. *Q.E.D.*

For any distribution of types implied by $f(\cdot)$, any real interest rate r, and any time interval between periods z, let $\Sigma(f, r, z)$ denote the set of sequential equilibria of the alternating-offer game which satisfy Assumptions A.1–A.4. For

$0 < M \leqslant 1 \leqslant L < \infty$ and $0 < \alpha < \infty$, let $\mathscr{F}_{L,M,\alpha}$ denote the set of all functions $f(\cdot)$ such that $M(1-q)^{\alpha} \leqslant f(q) \leqslant L(1-q)^{\alpha}$ for all $q \in [0,1]$.[27] This notation permits us to state a theorem which is closely related to the main theorem of Gul and Sonnenschein (1988), but instead treats the case of "no gap" and establishes the same type of uniformity as in Theorem 5.4 of Ausubel and Deneckere (1989a). Our theorem states that, as the time interval between offers approaches zero, the introductory price converges to the seller's valuation (i.e., zero), uniformly over all valuation functions in the set $\mathscr{F}_{L,M,\alpha}$.

THEOREM 3.2 (THE ALTERNATING-OFFER, UNIFORM COASE CONJECTURE): *For every* $0 < M \leqslant 1 \leqslant L < \infty$, $0 < \alpha < \infty$, *and* $\varepsilon > 0$, *there exists* $\bar{z}(L, M, \alpha, \varepsilon) > 0$ *such that for every* $f \in \mathscr{F}_{L,M,\alpha}$, *for every* z *satisfying* $0 < z < \bar{z}(L, M, \alpha, \varepsilon)$, *and for every equilibrium belonging to* $\Sigma(f, r, z)$, *the initial serious (seller or buyer) offer is less than or equal to* ε.

PROOF: See Appendix.[28]

The intuition for the Coase Conjecture and the outline of its proof is as follows: If the introductory price were high, then some substantial real time would have to elapse before the price became low (or else a rational buyer would wait to purchase). When the time between offers is made negligible, this means that many successive prices are very close together, so that the seller is excessively price discriminating. Given the stationarity and monotonicity assumptions, the seller has the opportunity to *accelerate* sales from later periods to earlier periods. In the formal proof, we show that the gains from accelerated trade eventually exceed the losses from diminished price discrimination, implying that the seller could profitably deviate from the equilibrium, and generating a contradiction.

An important observation should be made concerning Lemma 3.1 and Theorem 3.2. Kreps and Wilson's (1982) definition of sequential equilibrium for finite games implies that whenever (after any history) the seller revises her beliefs about the buyer, she posits a new distribution function that has its support entirely contained in the support of the prior distribution of buyer types (i.e., the support of $F(\cdot)$). While it may be desirable to also impose this restriction on equilibria of infinite games, the proofs of Lemma 3.1 and Theorem 3.2 do not depend on such a restriction and, therefore, we have not made such a restriction in the current paper. Rather, we allow the seller's beliefs to wander outside

[27] The reader may wonder whether there exist *any* valuation functions for the case of "no gap" which are not elements of $\mathscr{F}_{L,M,\alpha}$. The two simplest examples we know are $f(q) = \exp(-q/(1-q))$ and $f(q) = 1/(1 - \log(1-q))$.

[28] The proof of the theorem draws heavily on Gul, Sonnenschein, and Wilson (1986), and Gul and Sonnenschein (1988). We learned a lot from these authors, and are glad to be able to acknowledge our intellectual debt here.

the initial support of buyer valuations, after histories which have zero probability of occurrence.[29] In the proof of Theorem 3.3, below, this enables us to restate a version of Theorem 3.2 which holds at the start of *any* (as opposed to just the initial) period following any history in which no prior buyer deviations have occurred (and where the seller may have narrowed her beliefs so that some high-valuation buyer types have zero posteriors).

With our intermediate results in hand, we may now prove the main theorem:[30]

THEOREM 3.3 (THE SILENCE THEOREM): *Let f belong to $\mathscr{F}_{L,M,\alpha}$ and let r be any positive interest rate. Then there exists $\bar{z} > 0$ such that, whenever the time interval between offers satisfies $0 < z < \bar{z}$ and for every equilibrium belonging to $\Sigma(f, r, z)$, the informed party never makes any serious offers in the alternating-offer bargaining game, both along the equilibrium path and after all histories in which no prior buyer deviations have occurred.*

PROOF: We begin by demonstrating that a version of Theorem 3.2 also holds after all histories with no prior buyer deviations: there exists $\bar{z} > 0$ such that for every z $(0 < z < \bar{z})$ and for every $(\bar{\sigma}, \bar{g}) \in \Sigma(f, r, z)$, the next serious offer *after a state of q* is at most $\varepsilon f(q)$, where q is any state entering an even-numbered period after any history without prior buyer deviations induced by $(\bar{\sigma}, \bar{g})$. The proof is as follows. Let $f_q(\cdot)$ denote the *rescaled* residual valuation function from $f(\cdot)$ when the state is $q \in [0,1)$, i.e., $f_q(x) = f[q + (1-q)x]/f(q)$, for all $x \in [q, 1]$. As in Lemma 5.3 of Ausubel and Deneckere (1989a), if $f \in \mathscr{F}_{L,M,\alpha}$, then $f_q \in \mathscr{F}_{L',M',\alpha}$, where $L' \equiv L/M$ and $M' \equiv M/L$. At the same time, let $(\bar{\sigma}_q, \bar{g}_q)$ denote the continuation of $(\bar{\sigma}, \bar{g})$ from the time that the state reaches q. Importantly, observe that when all prices and valuations in $(\bar{\sigma}_q, \bar{g}_q)$ are appropriately rescaled upward (via multiplication by $1/f(q)$), $(\bar{\sigma}_q, \bar{g}_q)$ becomes a

[29] Fudenberg and Tirole's (1988) "perfect Bayesian equilibrium," defined for finite games, also permits a type with zero prior probability to attain a positive posterior probability following an out-of-equilibrium history.

[30] A weaker version of the Silence Theorem also holds for the case of "the gap." Let $F(\cdot)$ be a distribution function whose support is the interval $[\underline{b}, \bar{b}]$, $0 < \underline{b} < \frac{1}{2}\bar{b}$, with the property that $F(\cdot)$ is continuous in a neighborhood of $\underline{b}$. Then for any $\tilde{b} \in (2\underline{b}, \bar{b})$, there exists $\bar{z} > 0$ such that in any equilibrium with time interval z $(0 < z < \bar{z})$, the buyer never reveals that his type is $\geqslant b$, for any $b \geqslant \tilde{b}$.

This result in essence establishes that, for small z, the only messages that the buyer ever sends in equilibrium are that his type is $\geqslant b$, for $b \in [\underline{b}, 2\underline{b}]$. Observe that the silence result is continuous as we go from a small gap to no gap: as $\underline{b}$ approaches zero, the interval of permissible messages collapses to the singleton message "my type is $\geqslant 0$." This is the only message transmitted in the case of no gap; the message is obviously vacuous.

It may not be clear to the reader why a literal Silence Theorem fails in the case of a gap. The simplest reason is that, under appropriate distributional assumptions, the bargaining has a final period $N < \infty$ (Fudenberg, Levine, and Tirole (1985)). Suppose that the informed party makes no serious offers, that the truncated support entering the last period is $(q_N, 1]$, and that the uninformed party makes the final offer, p_N. Then it may also be an equilibrium for *all* remaining types of the uninformed party to offer p' in period $N-1$, where valuation $f(q_N)$ is indifferent between p' today and p_N tomorrow. (In contrast, in the case of no gap, there is no final period.)

sequential equilibrium for valuation function f_q (i.e., $(\bar{\sigma}_q, \bar{g}_q) \in \Sigma(f_q, r, z)$).[31] Thus, for positive $z < \bar{z}(L', M', \alpha, \varepsilon)$, Theorem 3.2 implies that the initial serious offer in $(\bar{\sigma}_q, \bar{g}_q)$ is at most ε. By rescaling, the next serious offer in $(\bar{\sigma}, \bar{g})$ after q is at most $\varepsilon f(q)$.

We may now easily establish the Silence Theorem. Suppose that the theorem did not hold. Then for any $\bar{z} > 0$, there would exist $f \in \mathscr{F}_{L,M,\alpha}$, positive time interval $z < \bar{z}$, sequential equilibrium $(\bar{\sigma}, \bar{g}) \in \Sigma(f, r, z)$, and buyer types q and q' $(0 \leqslant q < q' < 1)$ with the property that, at some point in the equilibrium (or after some history without prior buyer deviations), the interval $(q, q']$ of buyers makes a serious counteroffer. To be more precise, there is an odd-numbered period j such that, along a history without buyer deviations, the set of buyers remaining at the start of period j is $(q, 1]$. In period j, the buyers partition into two nondegenerate subintervals as follows: buyers in $(q, q']$ make a serious counteroffer p; whereas buyers in $(q', 1]$ make a nonserious counteroffer.

We now will show that buyer q' can profitably deviate by mimicking $(q', 1]$ in making a nonserious counteroffer.[32] Suppose that q' follows the prescribed equilibrium strategy. Since q' reveals himself to be contained in $(q, q']$ when he offers p, the seller immediately comes to maintain beliefs that the valuation of q' is at least $f(q')$. Since p is defined to be a serious counteroffer, it must be accepted by the seller; by Lemma 3.1, $p \geqslant (\delta/(1+\delta))f(q')$. Hence, the payoff (evaluated in period j) to q' from equilibrium play equals $f(q') - p$, which is bounded above by $(1/(1+\delta))f(q')$. Alternatively, q' may deviate by making a nonserious counteroffer. This deviation is undetectable and, hence, the state entering period $j+1$ equals q'. By stationarity, the next serious offer must occur in either period $j+1$ or $j+2$. Let $\bar{z}$ be any positive time interval less than $\bar{z}(L', M', \alpha, \frac{1}{4})$. By the version of Theorem 3.2 proven two paragraphs above, the next serious offer will be at most $\frac{1}{4}f(q')$. Hence, the payoff from deviating is bounded below by $\delta^2[f(q') - \frac{1}{4}f(q')] = \frac{3}{4}\delta^2 f(q')$. Let $\bar{z}$ also be chosen sufficiently small that $\delta \equiv e^{-rz}$ satisfies $\frac{3}{4}\delta^2 > (1/(1+\delta))$, whenever $0 < z < \bar{z}$. Then the payoff from deviating exceeds the equilibrium payoff, providing a contradiction. *Q.E.D.*

[31] Observe that this is the sentence of the proof which necessitated our discussion, earlier in this section, of *not* restricting sequential equilibria to have the property that the seller's revised beliefs be entirely contained in the support of the prior distribution of buyer types. If we had made that restriction in our definition of "sequential equilibrium," then it would not necessarily be the case that $(\bar{\sigma}_q, \bar{g}_q)$ is a sequential equilibrium for valuation function f_q. The reason is that $(\bar{\sigma}_q, \bar{g}_q)$ was derived from a sequential equilibrium $(\bar{\sigma}, \bar{g})$ for valuation function f. In $(\bar{\sigma}, \bar{g})$, after a history in which beliefs are entirely contained in $(q, 1]$, it is still possible (off the equilibrium path) for beliefs subsequently to be revised outside $(q, 1]$. This translates to beliefs $\bar{g}_q$ possibly wandering outside the support of the prior distribution from f_q.

With the more restrictive definition of sequential equilibrium, in order to prove the analogue to Theorem 3.3, it would be necessary to separately observe that Theorem 3.2 holds even if revisions outside the support of the prior are allowed. We believe the present solution of not restricting the definition of sequential equilibrium is superior on the grounds that it proves a stronger theorem, and is more elegant.

[32] Since q' will strictly prefer to deviate and $f(\cdot)$ is left-continuous, it is in fact the case that a *positive measure* of buyer types in $(q, q']$ can profitably deviate by mimicking types $(q', 1]$.

A "silence equilibrium" of the alternating-offer game with time interval of z between periods bears a close resemblance to an equilibrium of the seller-offer game with time interval of $2z$ between periods. We can demonstrate a precise equivalence between the equilibrium outcomes of the two games. Consider any stationary sequential equilibrium of the seller-offer game for any time interval $2z > 0$ between periods. By a variant on Theorem 5.4 of Ausubel and Deneckere (1989a), a uniform Coase Conjecture holds: for each $\varepsilon > 0$, there exists $\bar{z} > 0$ with the property that, for any $z \in (0, \bar{z})$, every seller offer is less than ε times the highest remaining buyer valuation. Now consider the alternating-offer game with time interval z between offers and specify the following description of strategies: In every even-numbered period, the seller makes the same offer as in the stationary sequential equilibrium of the seller-offer game (given the same beliefs); and each buyer type uses the same acceptance strategy. In every odd-numbered period, the buyer counteroffers zero and the seller rejects. If the buyer deviates by counteroffering a positive price, the seller revises his conjectures to point beliefs that the buyer's valuation equals the highest valuation which remained in the market at the time the buyer deviated. After such a deviation, the seller plays the strategy from the Rubinstein (1982) complete-information game, and the buyer optimizes accordingly. For time interval z sufficiently short, trading at ε times the highest remaining buyer valuation in the next period is more attractive than trading at $\delta/(1+\delta)$ times the highest remaining buyer valuation in the current period, deterring the buyer from speaking. It is straightforward to see that this construction yields a sequential equilibrium satisfying A.1–A.4. Thus, we have just argued that every stationary sequential equilibrium outcome of the seller-offer game with time interval $2z$ between periods can be embedded as a sequential equilibrium outcome (satisfying A.1–A.4) of the alternating-offer game with time interval z between periods, provided z is sufficiently small.[33]

Conversely, consider any A.1–A.4 equilibrium of the alternating-offer game. By Theorem 3.3, provided z is sufficiently small, the buyer remains silent even after histories in which the seller has deviated (provided that the buyer has not). Thus, it cannot be the case that the seller is deterred from deviating by the prospect of a subsequent unfavorable buyer offer; the buyer is expected to continue to make only nonserious offers. Thus, we are able to map the alternating-offer equilibrium to a seller-offer equilibrium by specifying the following strategies: in any period of the seller-offer game, the seller makes the same offer as in (even-numbered periods of) the alternating-offer game (given the same beliefs); and the buyer uses the same acceptance strategy. This clearly yields a stationary sequential equilibrium for the seller-offer game.

[33] Observe that this construction is similar to that of Fudenberg, Levine, and Tirole (1985, Section 5.4), Gul and Sonnenschein (1988, Section 4), and Ausubel and Deneckere (1989b, p. 34). However, unlike these previous constructions, the current construction satisfies the support restriction: in every period, the support of beliefs about the buyer's type is a subset of the support in the previous period, even *off* the equilibrium path.

Analogous to before, let $\Sigma'(f, r, 2z)$ denote the set of stationary sequential equilibria of the seller-offer bargaining game. We have thus shown the following theorem.

THEOREM 3.4: *Let f belong to $\mathscr{F}_{L,M,\alpha}$ and let r be any positive interest rate. Then there exists $\bar{z} > 0$ such that, whenever z satisfies $0 < z < \bar{z}$, the sets of equilibrium outcomes associated with $\Sigma(f, r, z)$ and $\Sigma'(f, r, 2z)$ exactly coincide.*

Consider arbitrary $f \in \mathscr{F}_{L,M,\alpha}$ and $r > 0$. By Theorem 4.2 of Ausubel and Deneckere (1989a), the existence of a stationary equilibrium of the seller-offer game is guaranteed for any time interval between offers. For $z \in (0, \bar{z})$, the above logic assures that $\Sigma(f, r, z)$ is also a nonvacuous set; the existence of "silence equilibria" in the alternating-offer game is guaranteed.

The Silence Theorem holds not only for the (1, 1)-alternating-offer game but, in fact, for all alternating-offer games in which k ($\geqslant 1$) seller offers are followed by l ($\geqslant 0$) buyer counteroffers. In (k, l)-alternating-offer games, the definition of stationarity is appropriately modified by requiring the buyer's offer behavior to depend only on the current state *and the period modulo* $(k + l)$, and the definition of monotonicity is modified similarly.[34] Let $\Sigma^{k,l}(f, r, z)$ denote the set of sequential equilibria of the (k, l)-alternating-offer game which satisfy stationarity, monotonicity, pure strategies, and no free screening. Then Theorem 3.2 continues to hold for $\Sigma^{k,l}(f, r, z)$. Meanwhile, recall that Lemma 3.1 required that, for $\delta \approx 1$, the seller reject counteroffers less than $\approx \frac{1}{2}f(\bar{q})$. Similarly, for the (k, l)-alternating-offer game, an analogue to Lemma 3.1 requires the seller to reject counteroffers less than $\approx (k/(k + l))f(\bar{q})$.[35] Hence, the logic behind the proof of Theorem 3.3 carries through for general (k, l):

THEOREM 3.5: *Let f belong to $\mathscr{F}_{L,M,\alpha}$ and let r be any positive interest rate. Let $k \geqslant 1$ and $l \geqslant 0$. Then there exists $\bar{z} > 0$ such that, whenever the time interval between offers satisfies $0 < z < \bar{z}$ and for every equilibrium belonging to $\Sigma^{k,l}(f, r, z)$, the informed party never makes any serious offers in the (k, l)-alternating-offer bargaining game, both along the equilibrium path and after all histories in which no prior buyer deviations have occurred.*

If $k = 0$, then we actually find ourselves in the game where the buyer makes all the offers. In Ausubel and Deneckere (1989b, Theorem 4), we proved that this game has a unique sequential equilibrium. All buyer types pool by making an offer demanding all the surplus; this offer is immediately accepted. The

[34] To be more precise, A.3 is modified to read: "For every n and m which are periods in which the buyer makes offers and such that $n \equiv m \pmod{k + l}$...." Similarly, A.4 is modified to read: "For every n and m which are periods in which the seller makes offers and such that $n \equiv m \pmod{k + l}$...."

[35] To be more precise, in period $n \equiv j \pmod{k + l}$, where $k \leqslant j \leqslant k + l - 1$, the seller will reject any counteroffer less than $[\delta^{k+l-j}(1 - \delta^k)/(1 - \delta^{k+l})]f(\bar{q})$. In period $n \equiv i \pmod{k + l}$, where $0 \leqslant i \leqslant k - 1$, the seller will not offer any price less than $[1 - \delta^{k-i}(1 - \delta^l)/(1 - \delta^{k+l})]f(\bar{q})$. See Ausubel and Deneckere (1989b, Theorem 5).

Silence Theorem no longer literally holds in this case; nevertheless, the informed party never reveals any information via his own offers.[36]

4. REMARKS

a. *Relationship with Finite-Horizon Bargaining Games*

In many complete-information environments, the stationary equilibria of infinite-horizon games correspond to limits of the equilibria of finite-horizon versions of the same game (provided that the latter equilibria are unique). For example, consider any stage game with a unique Nash equilibrium. Then the stationary equilibrium of the infinite-horizon supergame is the limit of the unique Nash equilibria of the corresponding finite-horizon supergames. Similarly, restrictions to Markov strategies in infinite-horizon dynamic games with Markov structures are partly motivated by reference to backward-induction equilibria of their finite-horizon counterparts. An analogous observation holds for the infinite-horizon bargaining game with one-sided incomplete information in which the uninformed party makes all the offers. Each finite-horizon equilibrium is necessarily stationary[37] and is generically unique. Moreover, the limit of the equilibria is a (stationary) equilibrium of the limit game.

The close connection between stationary equilibria of an infinite-horizon game and limits of equilibria of finite-horizon versions of the same game may be regarded as one of the best defenses (although, perhaps, also as one of the most biting criticisms) of stationarity. It is thus interesting to explore this relationship for the alternating-offer bargaining game.

Consider the family of distribution functions which are *invariant under rescaling*. Let $F(v) = v^{\alpha}$, for any $\alpha > 0$, so that the corresponding valuation function is $f(q) = (1-q)^{1/\alpha}$. Observe that the rescaled residual valuation function of $f(\cdot)$ at q is given by $f_q(x) = (1-x)^{1/\alpha}$; conditional distributions formed by truncation are merely rescaled versions of the initial distribution.

We obtain a rather striking result for the $2N$-period, alternating-offer bargaining game in which the uninformed party makes the last offer. For every $\alpha \in (0, \infty)$, there exists $\underline{\delta} < 1$ such that, for any discount factor between $\underline{\delta}$ and 1 and for any positive integer N, this $2N$-period bargaining game has a *unique* equilibrium outcome satisfying A.1–A.2. This equilibrium is again characterized by silence: in every period, when it is the informed party's turn to make an offer, he gives up his move by making a nonserious offer. Moreover, in the limit as N approaches ∞, the equilibrium becomes stationary, thereby automatically satisfying A.3–A.4. A proof is provided in the Appendix.[38]

[36] A result analogous to Theorem 3.4 also clearly holds for *all* (k,l)-alternating-offer games.

[37] For finite-horizon games, stationarity should be taken to mean that players' actions depend only on the current state *and the number of periods remaining in the game*.

[38] We conjecture that this result does not depend on the restriction to distribution functions which are invariant under rescaling, but rather that (generic) uniqueness and silence hold at least also for the set of all differentiable functions $f(\cdot)$ whose derivatives satisfy $0 < M \leqslant f' \leqslant L < \infty$. The reason for our belief is that the following, hitherto undiscovered, finite-horizon version of the

However, the relationship between stationary equilibria and limits of finite-horizon equilibria is not so intimate as the above discussion may suggest. Consider instead the $2N$-period, alternating-offer bargaining game in which the informed party makes the last offer. Then for $N \geqslant 2$ it is straightforward to demonstrate that: (1) the informed party is necessarily *not* silent in the third-to-last period of any sequential equilibrium; and (2) there is a multiplicity of equilibria satisfying A.1–A.2. This multiplicity, in turn, implies that the finite-horizon equilibria no longer need to be stationary and, consequently, the limits are not necessarily stationary.[39]

b. *Relationship with the Signaling Literature*

A bargaining game is in some ways reminiscent of a standard signaling game. When a privately-informed buyer makes low offers, he indicates his willingness to delay agreement and, hence, a low valuation. In games of this genre, the common wisdom holds that, under any forward-induction type of refinement,[40] separation will take place if it is at all possible.[41] Our equilibria, on the other hand, have the informed party always pool by making nonserious offers. One might therefore wonder whether our assumptions leave no room for this type of refinement, or whether we obtain the Silence Theorem because (for small discount rates) the cost of delay through nonserious price offers is sufficiently small that silence (i.e., pooling) follows from incentive compatibility.

Within the confines of Assumptions A.1–A.4, incentive compatibility is the driving force behind our pooling result. Observe that, in the bargaining game, the low buyer type is the strong type. High (i.e., weak) buyer types would like to mimic low types unless the cost of imitation (i.e., the cost of delay) is made sufficiently great. However, under A.1–A.4, Theorem 3.2 shows that the Coase

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uniform Coase Conjecture appears to hold: Consider the sequence of N-period, seller-offer bargaining games of incomplete information in which the real time that elapses from period 1 to period N is held fixed at T (i.e., $z = T/(N-1)$). Then, for every T and in every sequential equilibrium, the seller's initial price converges to $e^{-rT}p^*(f)$ as $N \to \infty$, where $p^*(f)$ denotes the monopoly price against $f(\cdot)$. [Note that: (a) this finite-horizon version of the Coase Conjecture has the same content as the standard Coase Conjecture when $T \to \infty$; and (b) the finite-horizon version can also be verified computationally for distribution functions which are invariant under rescaling.] Also consider the sequence of $2N$-period, alternating-offer bargaining games of complete information (where the seller's valuation equals zero and the buyer's valuation equals one) in which the real time that elapses from period 1 to period $2N$ is held fixed at T (i.e., $z = T/(2N-1)$) and in which the seller makes the last offer. As $N \to \infty$, the initial price converges to $\frac{1}{2}(1 + e^{-rT})$. It should now be observed that the buyer of valuation one prefers the former (silence) limiting price to the latter (speaking) limiting price: $e^{-rT}p^*(f) < \frac{1}{2}(1 + e^{-rT})$, for all $T \geqslant 0$. Since the specified set of functions $f(\cdot)$ is closed under truncation and rescaling, the highest remaining buyer type prefers the limiting silence price over the limiting speaking price in each subsequent period as well.

[39] The reader may liken this situation to a supergame in which the stage game has multiple equilibria. Similarly, equilibria of the finitely-repeated version no longer need to be stationary in the sense that history matters only insofar as the number of periods remaining; consequently, the limits are not necessarily stationary in the standard sense.

[40] Kohlberg and Mertens (1986), Kohlberg (1989).

[41] Banks and Sobel (1987), Cho and Kreps (1987), and Cho and Sobel (1990).

Conjecture holds and, hence, that delay costs are insignificant when the time interval between periods is relatively short. This explains why buyer offers are always completely pooling; since the lowest buyer type always makes nonserious offers (see footnote 9), pooling here is tantamount to silence.

More generally, simple incentive compatibility arguments show that *separation requires real delay*. Indeed, when the discount factor δ is relatively close to one, the (signaling) cost of delaying agreement by one period is too low to permit instantaneous full separation. For consider buyer types $q'' < q' < 1$. If q'' fully separates before trading, then q'' obtains equilibrium utility of at most $(1/(1+\delta))f(q'')$ (Lemma 3.1, above). However, if q' also separates in the initial period, then q' does no worse than trading at $(\delta/(1+\delta))f(q')$ two periods hence (Grossman and Perry (1986, Lemma 3.1(ii))). For arbitrary fixed $q'' < 1$, it is therefore always possible to select type q' $(q'' < q' < 1)$ with a lower valuation such that q'' would wish to mimic q'. Thus, incentive compatibility requires that full separation cannot occur in a single period. In fact, a similar argument shows that for any fixed, real time t (such that $e^{-rt} > \frac{1}{2}$), it is impossible to have full separation within time t even as the time interval between periods is made to approach zero.[42] The reason for this is quite intuitive: delay is the signaling device which enables separation, but once separation occurs, trade ensues almost immediately. This means that separation must occur slowly, over an extended period of time.

Since separation requires real delay, one must move away from the Coase Conjecture (and hence the joint hypotheses of A.1–A.4) to obtain active signaling in sequential equilibrium. The Coase Conjecture is intimately linked to the notion of stationarity and, hence, this would seem to be the most obvious hypothesis to relax. Indeed, we have shown elsewhere (Ausubel and Deneckere (1989b)) that nonstationary equilibria may support delay.

The incongruity of "separation" and "no delay" raises the issue of whether the Coase Conjecture itself is compatible with forward induction. A direct investigation of this issue in our model is problematic, as the extant definitions pertain only to finite-horizon models. But it is feasible to investigate the implications of forward induction for the finite-horizon versions of the alternating-offer bargaining games considered in Section 4a. For these finite-horizon games, one can formally demonstrate that Assumptions A.1–A.2 are inconsistent with forward induction.[43]

[42] If buyer types monotonically separate over time, this argument can be repeated to show that full separation cannot occur in *any* finite, real time.

[43] In any finite-horizon version in which the seller makes the last offer, observe that for every A.1–A.2 equilibrium in the case of "no gap" there are necessarily buyer types $q'' < q' < 1$ such that the interval (q'', q') purchases in the final period while the interval $(q', 1]$ does not trade at all. Clearly, the seller's offer, p, in the final period must equal $f(q')$. Consider now a deviating offer $p' \in (\delta p, p)$ by the buyer in the next-to-last period. The seller must reject this offer, for otherwise all buyer types q such that $q \geqslant q'$ but $f(q) > p'$ would strictly benefit relative to the equilibrium. Notice however that in order to justify rejecting p', the seller's counteroffer after this deviation must be at least p'/δ, which in turn is strictly greater than p. Consequently every buyer type $q \in [0, q')$ would be strictly worse off from such a deviation. By forward induction, we can eliminate all such type-message pairs. In the reduced game, however, any buyer type q such that $q \geqslant q'$ but $f(q) > p'$

However, this argument is inconclusive on several grounds; ultimately, the compatibility of the Coase Conjecture and forward induction must be considered an open question. First, it is likely that the hypothesis required for the Coase Conjecture may be weakened; in particular, the full strength of A.1–A.2 may not be needed.[44] Second, since our argument relies crucially on the presence of a final period, it is quite possible that forward-induction equilibria of long, finite-horizon bargaining games would exhibit silence in all but the last few periods. Finally, in the finite-horizon games, forward-induction type arguments eliminate not only silence equilibria, but also some equilibria with nice signaling structures. A resolution of this interesting question is hampered by the fact that the bargaining game lacks the usual monotonicity[45] and single-crossing properties[46] of well-understood signaling games.

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would then deviate by offering p', which would have to be accepted in the reduced game, thereby breaking the equilibrium.

In any finite-horizon version in which the buyer makes the last offer, every A.1–A.2 equilibrium in the case of "no gap" necessarily has buyer valuations $q'' < q' < 1$ such that the interval (q'', q') purchases in the next-to-last period while the interval $(q', 1]$ trades at zero in the last period. Clearly the seller's offer, p, in the next-to-last period must make q' indifferent between purchasing in the next-to-last or last periods, i.e., $f(q') - p = \delta f(q')$, implying $p = (1-\delta)f(q')$. A similar argument as above applies to a deviating offer $p' \in (\delta^2 p, \delta p)$ by the informed party in the *second-to-last* period, again showing that the equilibrium fails to survive forward induction.

[44] For example, it is probably possible to obtain the Coase Conjecture after replacing A.1 with a weaker assumption which still requires information revelation to occur in a convex fashion, but which permits the seller to use mixed acceptance strategies in response to serious buyer offers. Observe that a mixed acceptance strategy allows more separation to occur within each period, because it provides the seller (i.e., the uninformed party) with a continuum of responses to the buyer's (i.e., the informed agent's) offer. Pure strategies restrict the seller to merely a binary response (i.e., "yes" or "no").

[45] A signaling game is defined to be monotonic if, for every message, all sender types have the same preferences over the receiver's mixed best responses (Cho and Sobel (1990, Section 3)). The nonmonotonicity of many bargaining games derives from the fact that the sender (the buyer) has the option of rejecting the receiver's (the seller's) counteroffer. For example, consider a two-period bargaining model in which the buyer makes an initial offer, which the seller can either accept or follow with a final (take-it-or-leave-it) counteroffer. This game reduces to a formal signaling game, provided that the buyer's weakly dominated actions in his accept/reject decision are eliminated, and the seller's accept/reject decision and counteroffer are collapsed into a single move. Now suppose that the buyer sends a first-period "message" of offering to pay p, where $p < \delta$. Two available best responses for the seller are as follows. First, the seller may accept the buyer's offer of p with probability one; refer to this as response A. Second, the seller may reject with probability one and make a counteroffer of p', where $p' \geqslant \delta p$, with probability one; refer to this as response B. It should now be observed that high-valuation buyers (types q such that $f(q) > p$) strictly prefer response A to response B, whereas low-valuation buyers (types q such that $f(q) < p$) strictly prefer response B to response A. This observation follows from the fact that high-valuation buyers obtain positive utility from response A but encounter higher prices and later trade under response B. However, low-valuation buyers obtain negative utility from response A; under response B, they can reject the seller's counteroffer and thus assure themselves at least zero utility. This establishes nonmonotonicity.

[46] A signaling game is said to satisfy the "single-crossing" property if, whenever two message-response pairs yield the same utility to some type of sender and one message is greater than the other, then all higher types prefer to send the higher message (Cho and Sobel (1990, p. 392)). This condition is crucial in standard arguments for separation, as it guarantees that higher types are more willing to send higher signals than lower types. The failure of the single-crossing property in many bargaining games can again be traced to the buyer's option of rejecting the seller's counteroffer. Consider, as in the previous footnote, the two-period, alternating-offer, bargaining model in which the seller makes the final offer, and consider the following two message-response

5. CONCLUSION

In this article, we have proven versions of the Coase Conjecture and the Silence Theorem for alternating-offer bargaining games. Whereas the Coase Conjecture concerns the limit as the time interval between offers approaches zero, there is good reason to think that the Silence Theorem will hold quite far away from the limit and, in that sense, is a more robust result. The intuition for the greater robustness is that the Coase Conjecture establishes a bound, ε, on serious offers and speaks about ε converging to zero as the time interval between successive offers approaches zero. However, the Silence Theorem does not require ε to be close to zero, but (loosely speaking) only that ε be somewhat less than $\frac{1}{2}$, so that the buyer will prefer waiting for an offer to revealing his type. This suggests that the time interval between offers can be allowed to become fairly long before the buyer finds it in his interest to speak. In this Conclusion, we will explore how long the time interval between offers can become before the buyer relinquishes his right to remain silent.

Let the distribution function be given by $F(v) = v^{\alpha}$, for any $\alpha > 0$, so that the corresponding valuation function is $f(q) = (1-q)^{1/\alpha}$. Following the discussion of Section 4a, observe that conditional distributions formed by truncation are merely rescaled versions of the initial distribution. We will refer to these distributions as *invariant under rescaling*. For this family, it is natural to examine sequential equilibria which not only satisfy A.1–A.4 but also are themselves invariant under rescaling.[47] Additionally, it is sensible to restrict attention to equilibria in which the seller's beliefs are not permitted to wander outside the support $[0,1]$ of the prior distribution $F(\cdot)$.

For the purposes of this Conclusion, let us redefine "state" to now denote the highest remaining buyer *valuation*. Invariance under rescaling requires that the continuation strategies, starting from any equilibrium state, look the same. Therefore, all offers, counteroffers, acceptance functions, and value functions must be linear in the state. Moreover, either there exists a serious buyer counteroffer in every (odd-numbered) period or else the buyer never makes serious counteroffers in any period.

pairs: (A) the buyer offers p, which the seller accepts with probability α and follows with the counteroffer $p'' > \delta^{-1}p$ in the event she rejects; and (B) the buyer offers p', which the seller rejects for sure and follows with the counteroffer $\delta^{-1}p'$. Note that the utility from option A to a buyer with valuation b is given by $U(b, A) = \alpha(b - p)$, if $p \leqslant b \leqslant p''$; and $U(b, A) = \alpha(b-p) + \delta(1-\alpha)(b - p'')$, if $b \geqslant p''$. The graph of this function in $b - U$ space is piecewise linear, with a slope equal to the (discounted) probability of trade, which equals α for $p \leqslant b < p''$ and $\alpha + \delta(1-\alpha)$ for $b > p''$. The utility of option B to a buyer with valuation b is given by: $U(b, B) = \delta b - p'$, for all $b \geqslant \delta^{-1}p'$. Since the graph of the latter function is linear with slope equal to δ, the graphs of $U(\cdot, A)$ and $U(\cdot, B)$ will intersect *twice* provided that $\alpha < \delta$ and the prices are chosen appropriately. For example, if $\alpha = .5$, $\delta = .6$, $p = .4$, $p' = 4/15$, and $p'' = 8/9$, buyers with valuations b such that $2/3 < b < 1$ strictly prefer option B, whereas b such that $.4 < b < 2/3$ or $b > 1$ strictly prefer option A.

[47] Following the discussion of Section 4a, the reader should observe that the sequential equilibria which are invariant under rescaling have an additional attractive property. They are the only sequential equilibria of the infinite-horizon game which are limits of A.1–A.2 equilibria of the finite-horizon game where the uninformed party moves last.

Let us assume for the moment that there does exist a serious buyer counteroffer in every odd-numbered period. (This will be true when δ is sufficiently far from one.) The offer/counteroffer structure along the equilibrium path can be described as follows, using constants $0 < \phi, \gamma, \eta, \theta, \mu \leqslant 1$. When it is the seller's turn to offer and the support of remaining buyer valuations equals the interval $[0, x)$, the seller proposes a price of ϕx. Buyers in the subinterval $[\gamma x, x)$ accept, whereas buyers in $[0, \gamma x)$ reject. Of the rejecting buyer types, an upper subinterval $[\theta\gamma x, \gamma x)$ proposes a serious counteroffer of $\eta\theta\gamma x$ in the next period, whereas the lower subinterval $[0, \theta\gamma x)$ makes a nonserious counteroffer and awaits the seller's next offer. Finally, let $V([a,b))$ denote the seller's expected present value of continuing the game when it is her turn to move and the set of remaining buyer valuations is the interval $[a, b)$. Then $V([0,x))$ is given by μx, where $\mu \equiv V([0,1))$.

The following conditions on the parameters $\{\phi, \eta, \theta, \mu\}$ can easily be derived. First, a buyer of valuation $\theta\gamma x$ should be indifferent between proposing the counteroffer $\eta\theta\gamma x$ and awaiting the offer $\phi\theta\gamma x$ one period later, i.e.,

$$(5.1) \qquad 1 - \eta = \delta(1 - \phi).$$

Second, the counteroffer $\eta\theta\gamma x$ must make the seller indifferent between acceptance and continuing the game with beliefs $[\theta\gamma x, \gamma x)$:

$$(5.2) \qquad \eta\theta = \delta V([\theta, 1)).$$

Third, the seller must be optimizing when choosing $p = \phi x$:

$$(5.3) \qquad \mu \equiv V([0,1)) = \max_p \{p\Phi([v(p),1)) + \delta\eta\theta v(p)\Phi([\theta v(p), v(p))) + \delta^2\Phi([0,\theta v(p)))V([0,\theta v(p)))\},$$

where $v(p)$ denotes the solution to $v(p) - p = \delta[v(p) - \eta\theta v(p)]$ and where $\Phi([a,b))$ denotes the probability that the buyer's valuation is contained in $[a, b)$ (i.e., $\Phi([a,b)) = b^\alpha - a^\alpha$ whenever $0 \leqslant a \leqslant b \leqslant 1$).

Let $\underline{\delta}(\alpha)$ denote the critical value at which the buyer stops speaking in the class of equilibria we consider when the parameter is equal to α. Then for any $\delta < \underline{\delta}(\alpha)$, it must be the case that the subinterval $[\theta\gamma x, \gamma x)$ is nondegenerate, while for $\delta > \underline{\delta}(\alpha)$, the subinterval $[\theta\gamma x, \gamma x)$ is degenerate. Consequently, when $\delta = \underline{\delta}(\alpha)$, the above system of equations must yield a solution with $\theta = 1$. Note, then, that (5.2) reduces to $\eta = \delta V([1,1]) = (\delta/(1+\delta))$, Rubinstein's (1982) complete information solution.[48]

[48] In any sequential equilibrium, after any history in which the seller's beliefs are concentrated at the upper bound of the support and in which it is the seller's turn to move, she offers a price of $1/(1+\delta)$, which is accepted immediately. This follows from Lemma 3.1, above, and equation (16) of Ausubel and Deneckere (1989b).

TABLE I

CALCULATION OF $\underline{\delta}(\alpha)$, THE MAXIMAL DISCOUNT FACTOR, AND $\underline{z}(\alpha)$, THE MINIMAL TIME INTERVAL BETWEEN SUCCESSIVE PERIODS, SUCH THAT THE INFORMED PARTY EVER SPEAKS IN THE RESCALE-INVARIANT SEQUENTIAL EQUILIBRIUM, THE RATIO OF PROFITS AT $\underline{\delta}(\alpha)$ TO COMMITMENT PROFITS, AND THE EXPECTED DISCOUNTED PROBABILITY OF TRADE AT $\underline{\delta}(\alpha)$, WHEN $F(v) = v^{\alpha}$ AND THE REAL INTEREST RATE $r = 10\%$ PER YEAR.

α	$\underline{\delta}(\alpha)$	$\underline{z}(\alpha)$	Ratio of Profits at $\underline{\delta}(\alpha)$ to Commitment Profits	Discounted Probability of Trade at $\underline{\delta}(\alpha)$
.10	.78805	28.58 months	.75301	.15169
.25	.79891	26.94 months	.74265	.31022
.50	.81458	24.61 months	.72779	.47569
1	.83929	21.02 months	.70440	.64780
2	.87271	16.34 months	.67225	.78927
4	.90976	11.35 months	.63469	.88425
10	.95164	5.95 months	.58665	.95126

Also observe that ϕ is the maximizer of (5.3). Maximizing (5.3) and substituting $\theta = 1$ and $\eta = (\delta/(1+\delta))$ yields

$$(5.4) \qquad \phi = (1+\delta)^{-1}\{(1+\alpha)[1-\alpha\delta^2(1+\delta)\phi/(1+\alpha)]\}^{-1/\alpha}.$$

Finally, from (5.1), we see that $\phi = 1 - [\delta(1+\delta)]^{-1}$. Substituting this into (5.4) and rearranging terms yields

$$\omega(\delta) \equiv \alpha\delta[\delta(1+\delta)-1]^{1+\alpha} - (1+\alpha)[\delta(1+\delta)-1]^{\alpha} + \delta^{\alpha} = 0.$$

Any solution to $\omega(\delta) = 0$ must satisfy $\delta(1+\delta) - 1 > 0$, that is, $\delta > \frac{1}{2}[\sqrt{5} - 1]$. Numerical simulations reveal that $\omega(\cdot)$ has a unique zero in $(\frac{1}{2}[\sqrt{5} - 1], 1)$; this zero is tabulated for various α in Table I.[49]

The numbers in Table I should be interpreted as follows. Suppose that the distribution function $F(\cdot)$ is linear and that the real interest rate is 10% per year. Then $\underline{\delta} = .83929$ is equivalent to saying that, in the sequential equilibrium which is invariant under rescaling, the informed party exercises his right to remain silent whenever the time interval between successive periods is less than $1\frac{3}{4}$ years. Since the extensive form has the parties alternate in making offers, this means that the buyer is silent unless each party has an opportunity to speak less than once every $3\frac{1}{2}$ years! As $F(\cdot)$ becomes arbitrarily concave, the requisite time interval between offers expands to a limiting value of 29.80 months; as $F(\cdot)$ becomes arbitrarily convex, the requisite time shrinks at a very slow rate toward zero. Even with the rather skewed distribution function

[49] Wilson (1987) tabulates the parameters of the Grossman-Perry (1986) equilibrium at various values of δ, for the case of the uniform distribution and assuming that the serious buyer counteroffer takes the form $\eta\theta\gamma x = [\delta/(1+\delta)]\theta\gamma x$. While there seems to be no justification for this assumption (other than simulations), Wilson found the same critical discount factor of .83929. This should not come as a surprise to the reader. First, the Grossman-Perry equilibrium satisfies rescaling invariance. Second, at the critical δ, the subinterval $[\theta\gamma x, \gamma x)$ collapses to a single point and, then, Grossman-Perry's support restriction justifies the choice $\eta = \delta/(1+\delta)$.

$F(v) = v^{10}$, the buyer only speaks when the time interval between periods is greater than half a year.

There is a fairly simple intuition as to why, in Table I, the requisite time interval between periods is made shorter as the distribution function becomes more convex. The force which discourages the informed party from speaking is that making a serious counteroffer would reveal that he has one of the highest remaining possible valuations. However, if the distribution function is very convex, then the uninformed party *already* places a high probability on the event that the informed party has one of the highest remaining possible valuations. (For example, if $F(v) = v^{10}$, the buyer's *ex ante* expected valuation is already 0.909.) In other words, even when the distribution function is made extraordinarily convex, the informed party retains the right to remain silent. Unfortunately, against an opponent who holds a sufficiently unfavorable prior distribution, silence becomes almost equally as incriminating as an admission of guilt.

The comparison between the Coase Conjecture and the Silence Theorem is all the more striking when we examine the seller's equilibrium profits. A useful index of the extent to which the Coase Conjecture bites at a given discount factor is the ratio of equilibrium profits to "commitment profits" (i.e., the profits which the seller could earn if she were able to precommit to a price path at the start of the game). Commitment profits are easily computed for any distribution of buyer valuations, as they equal the static monopoly profits against that distribution (Stokey (1979)). As δ approaches zero, the ratio of equilibrium profits to commitment profits converges to one (since the second period is pushed indefinitely far into the future); as δ approaches one, the ratio converges to zero.

In the fourth column of Table I, we report the ratio of equilibrium profits to commitment profits, computed at the critical discount factor which makes the Silence Theorem hold. Observe that the seller's profits there are far removed from those conjectured by Coase. At "ordinary" parameter values (e.g., $\alpha \approx 1$), silence prevails even while the seller earns fully 70% of the commitment profits. Even if α equals 1000, the ratio of profits at $\underline{\delta}(\alpha)$ to commitment profits is .50299; in the limit as α approaches infinity, the profit ratio converges to $\frac{1}{2}$. Thus, regardless of α, the seller exercises significant market power at discount factors which make silence mandatory for the buyer.

Finally, the Coase Conjecture can be rephrased as saying that one-sided incomplete information does not cause delay in bargaining (Gul and Sonnenschein (1988)). Hence, another good index of the extent to which the Coase Conjecture bites at a given discount factor is the expected discounted probability of trade, i.e., $\int e^{-rt(q)}\,dq$, where $t(q)$ denotes the time at which type q trades in a particular equilibrium. In the fifth column of Table I, we report this index, again computed at the critical discount factor which makes the Silence Theorem hold. For example, at $\alpha = 1$, the expected discounted probability of trade is only 64.78%; this compares with 50% under unconstrained monopoly pricing and 100% under the full Coase Conjecture. Even at $\alpha = 10$, the expected discounted probability of trade (95.13%) is closer to that of unconstrained monopoly pricing (90.91%) than to that of the Coase Conjecture. Thus, silence prevails even while there remains substantial real delay.

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Manuscript received September, 1989; final revision received August, 1991.

APPENDIX

Proof of Theorem 3.2

For any sequential equilibrium $(\bar{\sigma}, \bar{g})$, define the *effective price function* $\mathscr{P}$: $[0,1] \to [0,1]$ to associate with every buyer $q \in [0,1]$ the price $\mathscr{P}(q)$ he pays to the seller in the equilibrium. (Under the assumption of "no gap," the buyer $q = 1$ never purchases; for convenience, always define $\mathscr{P}(1) = 0$.) To be more precise, if the interval $(q^k, q^{k+1}]$ of buyers purchases at price p^k in equilibrium $(\bar{\sigma}, \bar{g})$, we will say $\mathscr{P}(q) = p^k$ for all $q \in (q^k, q^{k+1}]$. Without loss of generality, we will assume that $\mathscr{P}(\cdot)$ is left continuous and nonincreasing (see Ausubel and Deneckere (1989a, p. 516)).

Suppose that Theorem 3.2 does not hold. Then there exists $\varepsilon > 0$, a sequence $\{f_n\}_{n=1}^{\infty} \subset \mathscr{F}_{L,M,\alpha}$, a sequence of positive time intervals $\{z_n\}_{n=1}^{\infty} \downarrow 0$, and a sequence of stationary equilibria $\{\bar{\sigma}_n, \bar{g}_n\}_{n=1}^{\infty}$ (with effective price functions $\{\mathscr{P}_n\}_{n=1}^{\infty}$) such that $\mathscr{P}_n(0) > \varepsilon$ for all $n \geq 1$. Without loss of generality, we may assume that $\{\mathscr{P}_n\}_{n=1}^{\infty}$ converges pointwise for all rationals in $[0,1]$. (This can be assured by taking successive subsequences and applying a diagonal argument.) For every rational $r \in [0,1]$, let $\Phi(r) = \lim_{n \to \infty} \mathscr{P}_n(r)$. Define $\mathscr{P}(0) = \Phi(0)$ and, for every $x \in (0,1]$, define $\mathscr{P}(x) = \lim_{k \to \infty} \Phi(r_k)$, where each r_k is rational and $r_k \uparrow x$. Observe that $\mathscr{P}(\cdot)$ is well defined, left continuous, and nonincreasing. In the second part of the proof, we will demonstrate that the $\mathscr{P}(\cdot)$ so constructed is necessarily continuous. Let us assume this fact for the moment and show that the supposition of $\mathscr{P}_n(0) > \varepsilon$ leads to a contradiction.

Assuming the continuity of $\mathscr{P}(\cdot)$, select $\varepsilon', \varepsilon''$ and rational x_1, x_2 satisfying $0 < x_1 < x_2 < 1$ and $\mathscr{P}(0) \geq \varepsilon > \varepsilon' > \mathscr{P}(x_1) \geq \mathscr{P}(x_2) > \varepsilon'' > 0$. By construction, there exists $\hat{n}$ such that $\mathscr{P}_n(0) > \varepsilon$, $\mathscr{P}_n(x_1) < \varepsilon'$, and $\mathscr{P}_n(x_2) > \varepsilon''$ for all $n \geq \hat{n}$.

Define $t > 0$ such that buyer $q = 0$ is indifferent between a price ε at time zero and a price ε' at time t:

$$f_n(0) - \varepsilon = e^{-rt}[f_n(0) - \varepsilon'], \tag{A.1}$$

where $f_n(0) = 1$.

Observe that, in every equilibrium $(\bar{\sigma}_n, \bar{g}_n)$ $(n \geq \hat{n})$, no offer less than ε' can be accepted until after time t. (Otherwise, buyer $q = 0$, who purchases at time zero or later, would regret his purchase.) Since price has not dropped below ε' at time t, all buyers $q \geq x_1$ remain in the market at time t.

Following Gul and Sonnenschein (1988), we will now specify an "accelerated strategy" for the seller which compresses sales from time interval $[0, t]$ into the shorter interval $[0, t/2]$. For every n, define $N = \lfloor t/4z_n \rfloor$. Restrict attention to $n \geq \bar{n} \geq \hat{n}$, with $\bar{n}$ defined so that $N > 1$. Let $I^j = [1 - (j/N)(1 - \varepsilon'), 1 - ((j-1)/N)(1 - \varepsilon'))$, for $j = 1, \dots, N$. For each such j, select the smallest serious price offered in $\bar{\sigma}_n$ before time t which is contained in I^j (if one exists). Denote the resulting sequence of descending prices $p^1, \dots, p^m$ $(1 \leq m \leq N)$ and refer to these as the *good prices*. For each i $(1 \leq i \leq m)$, let $(q_1^i, q_2^i]$ denote the interval of buyers who purchased at p^i in $\bar{\sigma}_n$.

In the play of the accelerated strategy, let k denote the current period and q^k denote the current state, i.e., the set of buyers remaining at the start of period k is $(q^k, 1]$. Our objective is to induce each of the states q_2^i $(1 \leq i \leq m)$ in at most the first $2i$ periods. When $q^k \geq q_2^m$, the seller will have completed the acceleration phase and continues by inducing exactly the same states as in the original strategy from $\bar{\sigma}_n$.

We now describe the accelerated strategy for $q^k < q_2^m$. Define $i(k) = \min\{i \leqslant m: q_2^i > q^k\}$. The following prices, $\hat{p}^k$ and O^k, can be defined if q^k is an equilibrium state arising from $(\bar{\sigma}_n, \bar{g}_n)$.[50] If $p^{i(k)}$ was named by the *seller* in the original equilibrium, let $\hat{p}^k$ be a seller offer which induces a state of $q_2^{i(k)}$ when the current state is q^k. If $p^{i(k)}$ was named by the *buyer* in the original equilibrium, let $\hat{p}^k$ be a seller offer which induces a state of $q_1^{i(k)}$ when the current state is q^k. (The existence of $\hat{p}^k$ is guaranteed by monotonicity; furthermore, $\hat{p}^k \geqslant p^{i(k)}$.) Also, let O^k be the serious buyer counteroffer when the state is q^k, if a serious counteroffer exists; otherwise, define $O^k = 1$. The seller's strategy when $q^k < q_2^m$ is defined to be:

(A.2) If k is even:

- Offer $\hat{p}^k$.

If k is odd:

- Accept any counteroffer of at least O^k.
- Reject any lower counteroffer.

Observe, as in Gul and Sonnenschein (1988), that the following facts hold under the accelerated strategy:

(i) All trades (with any buyer type) which would have occurred in time interval $[0, t)$ under $(\bar{\sigma}_n, \bar{g}_n)$ now occur no later and at prices no more than $(1-\varepsilon')/N$ lower.

(ii) All trades (with any buyer type) which would have occurred in time interval $[t, \infty)$ under $(\bar{\sigma}_n, \bar{g}_n)$ now occur at least $t/2$ sooner and at the same or higher prices.

Statement (i) follows from the fact that the states induced under the accelerated strategy are a subsequence of the states under $(\bar{\sigma}_n, \bar{g}_n)$, and any trade which originally occurred at a price in interval I^j still occurs at a price in the same (or higher) interval. Statement (ii) follows from the fact that the state is brought beyond q_2^m in at most $2m$ periods and, hence, before a time of $2mz_n \leqslant 2Nz_n \leqslant t/2$. Monotonicity guarantees that all sales beyond q_2^m occur at the same or higher prices (but are accelerated by time $t/2$).

The acceleration strategy thus entails a loss in revenues from buyers $q \in [0, q_2^m]$ but provides a gain due to discounting from buyers $q \in (q_2^m, 1]$. Observe that $q_2^m < x_1$ for all $n \geqslant \bar{n}$. By (i), the monetary loss is less than $(1-\varepsilon')/N$ and the probability of loss is less than x_1. Hence, expected losses are bounded above by $x_1(1-\varepsilon')/N \leqslant [4x_1(1-\varepsilon')/(t-4z_n)]z_n$.

Let V denote the seller's expected payoff in $(\bar{\sigma}_n, \bar{g}_n)$ starting from when the state is q_2^m. V can be bounded below by $e^{-2rz_n}\varepsilon''[(x_2 - x_1)/(1-x_1)]$, as follows. Let $(q', q'']$ be the interval of buyers who purchase with x_2 at the price $\mathscr{P}(x_2)$. When the state is q_2^m and it is the seller's move, the seller has the following option: if $\mathscr{P}(x_2)$ was a seller (buyer) offer in $(\bar{\sigma}_n, \bar{g}_n)$, the seller charges a price $(\geqslant \mathscr{P}(x_2))$ which induces a state of q'' (q'). In the latter event, buyers in $(q', q'']$ reject the seller's offer and counteroffer $\mathscr{P}(x_2)$, which is accepted. This assures the seller an expected payoff of

$$e^{-rz_n}\mathscr{P}(x_2)\left[(q'' - q_2^m)/(1-q_2^m)\right] > e^{-rz_n}\varepsilon''\left[(x_2 - x_1)/(1-x_1)\right].$$

Meanwhile, when the state is q_2^m but it is the buyer's move, the seller can assure herself the same payoff, only discounted by one period, giving the desired lower bound.

The *ex ante* probability that the state will reach q_2^m is greater than $(1-x_1)$. The continuation profits upon reaching q_2^m are accelerated from a time not earlier than t to a time not later than $t/2$, and equal at least V. Hence, the expected gains are bounded below by

$$(e^{-rt/2} - e^{-rt})(1-x_1)V > (e^{-rt/2} - e^{-rt})e^{-2rz_n}\varepsilon''(x_2 - x_1).$$

Since $\lim_{n\to\infty} z_n = 0$, the expected gains from acceleration exceed the expected losses for sufficiently large n, demonstrating that acceleration is a profitable deviation from $\bar{\sigma}_n$. Thus, our initial assumption that $\mathscr{P}(\cdot)$ was continuous (and $\mathscr{P}(0) > 0$) leads to a contradiction.

It remains to be shown that $\mathscr{P}(\cdot)$ is continuous. Suppose otherwise. Since $\mathscr{P}_n(q) \leqslant f_n(q) \leqslant L(1-q)^\alpha$, for all n and q, it follows that $\mathscr{P}(q) \leqslant L(1-q)^\alpha$ and hence that $\mathscr{P}(\cdot)$ is continuous at 1. Therefore, there exists x $(0 \leqslant x < 1)$ where $\mathscr{P}(\cdot)$ is discontinuous. Define $d = [\mathscr{P}(x) - \lim_{q \downarrow x}\mathscr{P}(q)]/3$, $\hat{\varepsilon} = \mathscr{P}(x) - d$, and $\varepsilon' = \mathscr{P}(x) - 2d$. If $x \neq 0$, select η $(0 < \eta < x)$ such that $x - \eta$ is rational and η' $(0 < \eta' < \min(\eta, 1-x))$ such that $x + \eta'$ is rational. Also, for any

[50] Observe that it is sufficient to specify the accelerated strategy only for states which arise if the buyer does not deviate from his equilibrium strategy.

$\gamma \in (0, \eta')$, choose rational $q_\gamma^h \in (x - \gamma/2, x)$ and rational $q_\gamma^l \in (x, x + \gamma/2)$. Meanwhile, if $x = 0$, merely select η' $(0 < \eta' < 1)$ such that η' is rational. Always choose $q_\gamma^h = 0$ and, for any $\gamma \in (0, \eta')$, choose rational $q_\gamma^l \in (0, \gamma/2)$. By construction, for each γ, there exists $\hat{n}_\gamma$ such that $\mathscr{P}_n(q_\gamma^h) > \hat{\varepsilon}$ and $\mathscr{P}_n(q_\gamma^l) < \varepsilon'$ for all $n \geqslant \hat{n}_\gamma$. Define t by

$$f_n(0) - \hat{\varepsilon} = e^{-rt}[f_n(0) - \varepsilon'], \tag{A.3}$$

where $f_n(0) \equiv 1$. Observe that, in every equilibrium $(\bar{\sigma}_n, \bar{g}_n)$ $(n \geqslant \hat{n}_\gamma)$ a time interval exceeding t must elapse from the moment that q_γ^h purchases until the moment that q_γ^l purchases. ((A.3) implies that $f_n(q_\gamma^h) - \mathscr{P}_n(q_\gamma^h) < e^{-rt}[f_n(q_\gamma^h) - \mathscr{P}_n(q_\gamma^l)]$.)

We will now specify an accelerated strategy for the seller which compresses all sales from time interval $(t_{n,\gamma}, t_{n,\gamma} + t]$ into the shorter interval $(t_{n,\gamma}, t_{n,\gamma} + t/2]$, where $t_{n,\gamma}$ denotes the time that q_γ^h trades in $(\bar{\sigma}_n, \bar{g}_n)$. Similar to the first part of the proof, define $N = \lfloor t/4z_n \rfloor - 1$ and restrict attention to $n \geqslant \bar{n}_\gamma \geqslant \hat{n}_\gamma$, with $\bar{n}_\gamma$ defined so that $N > 1$. Define intervals I^j $(j = 1, \ldots, N)$ exactly as before. For each j, select the smallest serious price offered in $\bar{\sigma}_n$ during time interval $(t_{n,\gamma}, t_{n,\gamma} + t]$ which is contained in I^j (if one exists). Denote the resulting sequence of descending prices $p^1, \ldots, p^m$ $(1 \leqslant m \leqslant N)$ and define intervals $(q_1^i, q_2^i]$ as before. We will now induce each of the states q_2^i $(1 \leqslant i \leqslant m)$ in at most the first $2i + 1$ periods after $t_{n,\gamma}$.

The seller's accelerated strategy is as follows: during time interval $[0, t_{n,\gamma}]$, use the original strategy from $\bar{\sigma}_n$. Beginning at time $t_{n,\gamma} + z_n$, and until the state reaches q_2^m, follow the strategy of (A.2). After the state has reached q_2^m, the seller continues by inducing the same states as in the original equilibrium $(\bar{\sigma}_n, \bar{g}_n)$.

Fact (i) from above now holds for time interval $(t_{n,\gamma}, t_{n,\gamma} + t]$. Fact (ii) now holds for time interval $(t_{n,\gamma} + t, \infty)$. Additionally, all trades which would have occurred in time interval $[0, t_{n,\gamma}]$ under $\bar{\sigma}_n$ occur identically under the accelerated strategy.

The accelerated strategy may entail a loss in revenues from buyers $q \in (q_\gamma^h, q_2^m]$ but provides a gain due to discounting from buyers $q \in (q_2^m, 1]$. Observe that $q_2^m < q_\gamma^l$ and, hence, the probability of loss is less than $(q_2^m - q_\gamma^h) < (q_\gamma^l - q_\gamma^h) < \gamma$. Hence, expected losses are bounded above by $\gamma(1 - \varepsilon')/N \leqslant [4\gamma(1 - \varepsilon')/(t - 8z_n)]z_n$, discounted from time $t_{n,\gamma}$.

Let V denote the seller's expected payoff in $(\bar{\sigma}_n, \bar{g}_n)$ when the state is q_2^m. V can be bounded below by

$$e^{-rz_n}(1 - e^{-rz_n})(M/2)[(1 - x)/2]^\alpha[(1 - x - \gamma)/(1 - x - \gamma/2)],$$

as follows. When the state is q_2^m, the seller has the option of waiting at most one period and offering a price of $(1 - e^{-rz_n})f_n((1 + x)/2) \geqslant (1 - e^{-rz_n})M[(1 - x)/2]^\alpha$. Buyers $q \in (q_2^m, (1 + x)/2]$ find this to be an offer they cannot refuse, since the payoff from acceptance dominates obtaining the good for free in the next period. The probability that $q \in (q_2^m, (1 + x)/2]$, conditional on a current state of q_2^m, equals $[(1 + x)/2 - q_2^m]/[1 - q_2^m] > (1/2)(1 - x - \gamma)/(1 - x - \gamma/2)$, yielding the desired lower bound.

Since q_2^m is reached with a probability of greater than $(1 - x - \gamma/2)$ and subsequent revenues are at least V and are accelerated by at least $t/2$, the expected gains are bounded below by $(e^{-rt/2} - e^{-rt})e^{-rz_n}(1 - e^{rz_n})(M/2)[(1 - x)/2]^\alpha(1 - x - \gamma)$, discounted from time $t_{n,\gamma}$. For sufficiently large n, note that $1 - e^{-rz_n} > (r/2)z_n$.

We conclude that there exist k_1, k_2 $(0 < k_1, k_2 < \infty)$ such that losses are less than $k_1\gamma z_n$ and gains are greater than $k_2 z_n$. Since γ can be made arbitrarily small, the expected gains can be made to exceed the expected losses, demonstrating that $\mathscr{P}(\cdot)$ cannot be discontinuous. *Q.E.D.*

Proof of the Finite-Horizon Result

The proof of silence and generic uniqueness of equilibria satisfying A.1 and A.2 in the finite-horizon game (in which the uninformed party makes the last move) depends on a comparison of two extensive forms: (1) a $2N$-period, alternating-offer game where the time interval between periods equals z (and in which the uninformed party makes the last move); and (2) an N-period game in which only the uninformed party makes offers and where the time interval between periods equals $2z$. It will be important for the subsequent logic to observe that if, in an equilibrium of the first extensive form, the informed party "speaks" for the last time in period $2n - 1$, then the equilibrium continuation beginning at period $2n$ corresponds to an equilibrium of the second extensive form beginning at period n (with the same seller beliefs).

Let $a^k_{2N,z}$ denote the equilibrium offer in period k of the $2N$-period, alternating-offer bargaining game of complete information in which the buyer's valuation equals one and the seller's valuation equals zero. It is easy to calculate that $a^k_{2N,z} = (\delta/(1+\delta))(1+\delta^{2N-k})$, if k is odd, and $a^k_{2N,z} = (1/(1+\delta))(1+\delta^{2N-k+1})$, if k is even, where $\delta \equiv e^{-rz}$. (As N approaches ∞, observe that these values converge to the Rubinstein (1982) offers.)

Let $s^k_{N,2z}(x)$ denote the (highest) equilibrium offer in period k of the N-period, seller-offer bargaining game of incomplete information in which the seller's current beliefs are that the buyer's type q is contained in the subinterval $(x, 1]$. The offer $s^k_{N,2z}(x)$ may be determined by backward recursion. We have the following proposition:

PROPOSITION A.1: *Suppose that for every n $(1 \leqslant n \leqslant N)$:*

$$a^{2n-1}_{2N,z} > 1 - \delta + \delta \sup_{x \in [0,1)} \{s^n_{N,2z}(x)/f(x)\}. \tag{A.4}$$

Then an A.1–A.2 *equilibrium outcome of the $2N$-period version of the alternating-offer bargaining game with incomplete information (in which the seller makes the last offer) exists, is generically unique, and is characterized by silence.*

PROOF: First, we demonstrate that if (A.4) is satisfied, then a "silence" equilibrium exists. Suppose that, at every time it is the buyer's turn to make an offer, all types pool and name a price of zero. Suppose also that the seller uses the following updating rule: if, at any stage, the seller's beliefs consist of the subinterval $(x, 1]$ and the buyer offers more than zero, the seller infers that $q = x$ (or slightly above) and never updates her beliefs thereafter. A buyer of type q, when contemplating whether to make an offer p in period $2n-1$, will refrain from doing so if

$$f(q) - f(x)a^{2n-1}_{2N,z} \leqslant \delta[f(q) - s^n_{N,2z}(x)], \tag{A.5}$$

since any counteroffer $p < f(x)a^{2n-1}_{2N,z}$ will be rejected by the seller. For every $q \in [x, 1)$, inequality (A.5) is implied by inequality (A.4).

Second, we argue that when (A.4) holds, any A.1–A.2 equilibrium must have the buyer making exclusively nonserious offers in all periods of the game. Suppose to the contrary that there exists an equilibrium of the $2N$-period game in which a serious offer is made, and let period $2n-1$ be the *latest* period in which a serious offer is made (after some history with no prior buyer deviation). Then there exists a nonempty subinterval $(q_{2n-1}, q_{2n}]$ of types who make a serious offer of p in period $2n-1$, while subinterval $(q_{2n}, 1]$ make nonserious offers in period $2n-1$ (and all subsequent periods). So we must have

$$f(q_{2n}) - p = \delta[f(q_{2n}) - s^n_{N,2z}(q_{2n})], \tag{A.6}$$

since the continuation is an equilibrium of the seller-offer game. Moreover, by the finite-horizon analogue to Lemma 3.1, $p \geqslant f(q_{2n})a^{2n-1}_{2N,z}$. Combining this inequality with equation (A.6) leads to a contradiction of (A.4).

We conclude that if (A.4) is satisfied, then every A.1–A.2 equilibrium involves silence, and such an equilibrium exists. The equilibrium path of offers (by the seller) and acceptances is also the equilibrium path from the corresponding game where the seller makes all the offers and thus is generically unique (see Fudenberg, Levine, and Tirole (1985)). *Q.E.D.*

We will now recursively calculate the (unique) sequential equilibrium of the finite-horizon, seller-offer bargaining game when $f(\cdot)$ belongs to the parametric family of valuation functions $f(q) = (1-q)^{1/\alpha}$, where $\alpha > 0$, in order that we may verify that inequality (A.4) holds for sufficiently short time intervals between periods. Let Q_n denote the state at the beginning of period n. Following essentially the same notation as Ausubel and Deneckere (1989a), let $P_n(Q_{n+1})$ denote the reservation price of buyer type Q_{n+1} in period n, let $R_n(Q_n)$ denote the seller's net present value of profits evaluated in period n, and let $T_n(Q_n)$ denote the highest buyer type sold to in period n. The following three equations recursively define $P_n(\cdot)$, $R_n(\cdot)$, and $T_n(\cdot)$, given $P_{n+1}(\cdot)$, $R_{n+1}(\cdot)$, and $T_{n+1}(\cdot)$:

$$P_n(Q_{n+1}) = (1-\delta^2)(1-Q_{n+1})^{1/\alpha} + \delta^2 P_{n+1}(T_{n+1}(Q_{n+1})), \tag{A.7}$$

$$R_n(Q_n) = \operatorname*{arg\,max}_{Q \in [Q_n, 1]} \Pi_n(Q_n, Q), \tag{A.8}$$

$$T_n(Q_n) = \max_{Q \in [Q_n, 1]} \Pi_n(Q_n, Q), \tag{A.9}$$

where $\Pi_n(Q_n, Q) \equiv \{(Q - Q_n)P_n(Q) + \delta^2(1-Q)R_{n+1}(Q)\}/(1-Q_n)$ and δ^2 has replaced the usual δ on account that the time interval between periods is now $2z$.

The functional forms for $P_n(\cdot)$, $R_n(\cdot)$, and $T_n(\cdot)$ can be recursively shown to be

(A.10) $$P_n(Q) = A_n(1-Q)^{1/\alpha}; \qquad R_n(Q) = B_n(1-Q)^{1/\alpha}; \qquad T_n(Q) = 1 - C_n(1-Q).$$

Using (A.7), (A.8), and (A.9), we obtain the following recursive relationships:

(A.11) $$C_n = \left(\frac{1}{1+\alpha}\right)\left(\frac{A_n}{A_n - \delta^2 B_{n+1}}\right)$$

(A.12) $$B_n = A_n C_n^{1/\alpha}(1-C_n) + \delta^2 B_{n+1} C_n^{1+1/\alpha},$$

(A.13) $$A_n = 1 - \delta^2 + \delta^2 A_{n+1} C_{n+1}^{1/\alpha}.$$

The terminal conditions are easily calculated to be: $A_N = 1$; $B_N = (\alpha/(1+\alpha))(1/(1+\alpha))^{1/\alpha}$; and $C_N = (1/(1+\alpha))$. We may then determine A_{N-1} by (A.13), C_{N-1} by (A.11), and B_{N-1} by (A.12), etc., yielding the sequence $\{A_n, B_n, C_n\}$.

Observe that the ratio $s^n_{N,2z}(x)/f(x)$ is independent of x and is given by $P_n(T_n(0)) = A_n C_n^{1/\alpha}$. For each N and n ($1 \leqslant n \leqslant N$), there then exists a value δ_n^N such that (A.4) is satisfied with equality; for $\delta \in (\delta_n^N, 1)$, (A.4) holds with strict inequality, and for $\delta < \delta_n^N$, (A.4) is violated. Given any fixed $\alpha > 0$, it can now be observed that $\delta_N^N < \delta_{N-1}^N < \cdots < \delta_2^N < \delta_1^N$, and $\lim_{N\to\infty} \delta_1 = \underline{\delta} < 1$. Consequently, for every $\delta \in [\underline{\delta}, 1)$, inequality (A.4) is satisfied for all n, and so the conclusion of Proposition A.1 holds.

REFERENCES

Admati, A., and M. Perry (1987): "Strategic Delay in Bargaining," *Review of Economic Studies*, 54, 345–364.

Ausubel, L., and R. Deneckere (1989a): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57, 511–531.

——— (1989b): "A Direct Mechanism Characterization of Sequential Bargaining with One-Sided Incomplete Information," *Journal of Economic Theory*, 48, 18–46.

Banks, J., and J. Sobel (1987): "Equilibrium Selection in Signaling Games," *Econometrica*, 55, 647–661.

Bulow, J. (1982): "Durable-Goods Monopolists," *Journal of Political Economy*, 90, 314–332.

Cho, I.-K. (1990): "Uncertainty and Delay in Bargaining," *Review of Economic Studies*, 57, 575–596.

Cho, I.-K., and D. Kreps (1987): "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 102, 179–221.

Cho, I.-K., and J. Sobel (1990): "Strategic Stability and Uniqueness in Signaling Games," *Journal of Economic Theory*, 50, 381–413.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15, 143–149.

Fudenberg, D., D. Levine, and J. Tirole (1985): "Infinite Horizon Models of Bargaining with One-Sided Incomplete Information," in *Game Theoretic Models of Bargaining*, ed. by A. Roth. Cambridge: Cambridge University Press, 73–98.

Fudenberg, D., and J. Tirole (1983): "Sequential Bargaining with Incomplete Information," *Review of Economic Studies*, 50, 221–247.

——— (1988): "Perfect Bayesian and Sequential Equilibria: A Clarifying Note," mimeo, MIT.

Grossman, S., and M. Perry (1986): "Sequential Bargaining under Asymmetric Information," *Journal of Economic Theory*, 39, 119–154.

Gul, F., and H. Sonnenschein (1988): "On Delay in Bargaining with One-Sided Uncertainty," *Econometrica*, 56, 601–612.

Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155–190.

Harsanyi, J. C., and R. Selten (1988): *A General Theory of Equilibrium Selection in Games*. Cambridge, Mass.: MIT Press.

Kalai, E., and D. Samet (1985): "Unanimity Games and Pareto Optimality," *International Journal of Game Theory*, 14, 41–50.

Kohlberg, E. (1989): "Refinement of Nash Equilibrium: The Main Ideas," Harvard Business School Working Paper No. 89-073.

KOHLBERG, E., AND J.-F. MERTENS (1986): "On the Strategic Stability of Equilibria," *Econometrica*, 54, 1003–1037.

KREPS, D., AND R. WILSON (1982): "Sequential Equilibria," *Econometrica*, 50, 863–894.

MADRIGAL, V., T. TAN, AND S. WERLANG (1987): "Support Restrictions and Sequential Equilibria," *Journal of Economic Theory*, 43, 329–334.

MASKIN, E., AND J. TIROLE (1988): "A Dynamic Theory of Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56, 549–570.

MYERSON, R. (1991): *Game Theory: Analysis of Conflict*. Cambridge, Mass.: Harvard University Press.

RUBINSTEIN, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–109.

——— (1985): "A Bargaining Model with Incomplete Information about Time Preferences," *Econometrica*, 53, 1151–1172.

SOBEL, J., AND I. TAKAHASHI (1983): "A Multi-Stage Model of Bargaining," *Review of Economic Studies*, 50, 411–426.

STOKEY, N. (1979): "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 93, 355–371.

——— (1981): "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12, 112–128.

WILSON, R. (1987): "Game-Theoretic Analyses of Trading Processes," in *Advances in Economic Theory*, ed. by T. Bewley. Cambridge: Cambridge University Press, 33–70.