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Pollution Aspects of Nuclear Energy Use

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Models are developed wherein the society, wishing to maximize the present discounted value of energy output net of pollution costs and with fixed resources for energy production, must decide what portion of its energy needs will be met by fossil fuel and what portion by nuclear energy. It is shown that the pollution effects of each alternative are as important in determining the pattern of energy use as the cost of production of each alternative. Several sets of assumptions about production costs and pollution generation are considered.

1. INTRODUCTION

In the theory of exhaustible resources, the effect of resource substitutes on the optimal rate of depletion has been studied under a variety of assumptions. Dasgupta and Heal [2], for example, have examined the possibility that an inexhaustible substitute replaces the nonrenewable resource at an uncertain future date. Kamien and Schwartz [5] and Dasgupta *et al.* [3] have extended this model to allow investment in R & D to affect the time at which the substitute arrives. Smith [8], on the other hand, has examined a case in which an inexhaustible substitute is initially available, but at a high cost, and focuses on the sequence in which society will produce the nonrenewable resource and the renewable alternative.

One situation to which Smith's model applies is the choice between fossil fuel and solar energy—or breeder fission—as sources of electric power. The present paper also examines the choice between exhaustible and inexhaustible energy sources but emphasizes the pollution costs of energy alternatives.

Pollution considerations are particularly important in choosing between fossil fuels and breeder fission. The burning of fossil fuels discharges sulfur dioxide and particulate matter into the environment with adverse health and amenity effects. The operation of the breeder reactor may lead to three types of pollution problems [1]. Least serious is the routine emission of gases such as tritium and krypton from the power plant and fuel reprocessing facilities. These gases decay fairly rapidly and are in this sense analogous to the air pollution associated with fossil fuel power plants. A more serious problem is the release of long-lived radioactive isotopes, such as plutonium 239 (half-life = 24,000 years) and iodine 129 (half-life = 17,000,000 years), which are present in spent fuel. These substances may escape into the biosphere when the used fuel is transported from the reactor to reprocessing facilities, or when reprocessed plutonium is returned to power plants. Nuclear materials which cannot be reused are another source of pollution if not properly disposed of. Finally, there is the problem of a catastrophic accident resulting from sabotage or loss of control of a power plant or reprocessing facility.

Radioactive isotopes once they have escaped into the biosphere may remain suspended in the atmosphere or enter the soil or water. In any case they are not

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easily recaptured. This feature, combined with the slow decay rate of plutonium, implies that nuclear pollution must be viewed as a stock which, at least in historical time, is nondecreasing. The production of nuclear energy will therefore entail a user cost similar to the user cost of fossil fuel which should increase over time as the stock of pollution increases. If the benefits from producing an additional kilowatt hour of energy are finite, then the environmental user cost of nuclear energy must eventually exceed the marginal benefits of energy production. This implies that just as there is a limit to the stock of fossil fuel which can be extracted, there is an upper bound to the stock of nuclear waste which can be produced.

Whether this upper bound ultimately limits the production of nuclear energy depends on the extent to which radioactive emissions can be contained by allocating resources to pollution control. Proponents of nuclear power plants stress the fact that radioactive emissions could effectively be reduced to zero if a sufficient fraction of energy resources were devoted to nuclear safety. In this view, breeder reactors are capable of providing an inexhaustible energy supply. Critics of the breeder, however, believe that under present technology there will always be a significant probability of releasing nuclear waste regardless of the amount of resources devoted to pollution control. In this view a stock of radionuclides must build up, however slowly, which will eventually limit the use of nuclear energy as an alternative to fossil fuel.

Below we examine two models of nuclear energy production which reflect both optimistic and pessimistic views of nuclear technology. The first model assumes that society has allocated a fixed amount of resources to energy production and examines how these resources will be divided between the production of energy from fossil fuel and the production of nuclear energy. Here the amount of resources devoted to nuclear safety is not subject to choice and an irreducible amount of nuclear pollution is generated in proportion to energy output. Accumulation of this waste eventually causes society to cease production of nuclear energy. In the second model resources allocated to nuclear safety are subject to control and it is possible, for any given amount of resources, to produce some amount of nuclear energy without increasing the stock of radioactive pollution.

A key question in both models is what effect breeder fission has on the rate of depletion of fossil fuels. In a model in which a high-cost, nonpolluting alternative to fossil fuel is available, one would expect society to rely initially on fossil fuel and switch gradually to the nonpolluting alternative as the user cost of oil and natural gas rose. The use of fossil fuels would thus decrease over time. This result might be expected to carry over to the present case if the opportunity cost of using fossil fuel, i.e., the cost of nuclear energy production, remained constant over time. The environmental cost of nuclear fuel, however, will change as the stock of waste increases. Thus, depending on the relationship between the two user costs, nuclear fuel may initially be more attractive than fossil fuel and resources devoted to fossil production may increase over time as radioactive pollution builds up and nuclear energy becomes a less attractive alternative.

Whether the use of fossil fuel increases or decreases through time should also depend on the initial cost of fossil fuel production compared with the initial cost of nuclear energy production. Two extreme cases may be distinguished. The first, and possibly the most realistic, is the case where fossil fuel is initially cheaper to produce than nuclear energy. Here one would expect that society would at first devote most of its resources to fossil fuel, the cheaper alternative, switching to breeder reactors as the user cost of fossil fuel rose. The opposite case, in which breeder fission is initially cheaper, is less likely but nevertheless possible if the cost of producing fossil fuel includes the social cost of air pollution. Here society is likely initially to devote its resources to nuclear energy production, with fossil fuel replacing breeder fission as the environmental cost of nuclear pollution grows. Eventually, however, nuclear power may again become an attractive alternative as the user cost of fossil fuel rises. A formal analysis of these two cases appears in Section 2, below.

Another question of interest is how the path of nuclear energy production changes over time. In both models total resources devoted to the breeder reactor are the complement of resources devoted to fossil fuel production, and will therefore move in the opposite direction. It is, however, of interest to know how nuclear resources will be divided between energy production and expenditure on pollution control. In a model in which the possibility of producing fossil fuel is ignored and resources must go either to nuclear energy production or to the containment of nuclear pollution, one would expect the proportion of resources devoted to energy production to decrease over time as the environmental cost of pollution rose. Whether this result continues to obtain when resources may also be devoted to fossil fuel production depends on the relative costs of fossil fuel versus nuclear energy and on the initial size of the nuclear waste stock. An analysis is provided in Section 3 below.

2. MODEL I—NUCLEAR POWER EXHAUSTIBLE

In the first model we assume that society has allocated a constant flow, $\bar{L}$, of some variable input to energy production. This input, which we shall call labor, may be used to produce energy from fossil fuel or energy from breeder fission. We assume that fossil fuel energy, E_1, is an increasing, concave function of labor input, L_1,

$$E_1 = g(L_1),$$

$$g(0) = 0, g'(L_1) > 0, g''(L_1) < 0, g'(0) < \infty,$$

and that energy production reduces the stock of fossil fuel, S, proportionately,

$$\dot{S} = -g(L_1).^1 \tag{1}$$

This ignores the effect of resource scarcity on the cost of energy production and also neglects the possibility of substituting labor for raw material input in the production of energy. It does, however, capture the notion that the supply of fossil fuel is bounded, while permitting us to focus on the complications introduced by nuclear pollution.

[1]This implicitly assumes that each kilowatt hour of energy requires a fixed raw material input and that the stock of fossil fuel, S, is measured in kilowatt hours.

Nuclear energy output, E_2, is likewise assumed to be an increasing, concave function of labor allocated to breeder fission,

$$E_2 = f(L_2),$$
$$f(0) = 0, f'(L_2) > 0, f''(L_2) < 0, f'(0) < \infty.$$

In model I neither the level of waste emitted by fossil fuel power plants (for a given E_1) nor the amount of radionuclides which escape into the environment (for a given E_2) are subject to control by the decisionmaker. Since air pollution from fossil fuel power plants depends only on the level of current output, the function $g(L_1)$ may be interpreted as including the social costs of SO_2 and particulates. In the case of the breeder reactor we assume that some amount of radioactive material will escape into the environment in proportion to energy output and that this waste will increase the stock of radioactive isotopes in the biosphere, W,

$$\dot{W} = \gamma f(L_2), \qquad \gamma > 0. \tag{2}$$

This assumption reflects the view that regardless of the resources devoted to nuclear safety, there will always be a positive probability of radioactive isotopes escaping into the environment as a result, say, of an accident in transporting spent fuel to reprocessing plants. If enough radioactive material is transported, probabilities may be interpreted as relative frequencies and the right-hand side of (2) as the relative frequency of an accident times the amount of fuel transported.

The stock of radionuclides in the environment yields disutility to society by causing cancer and other diseases. Although the incidence of cancer for a given stock of plutonium depends on the size and distribution of plutonium particles and on resuspension rates, we shall assume that the utility received from nuclear waste is a decreasing, concave function of the stock W. The function $V(W)$,

$$V'(W) \leq 0, \qquad V''(W) < 0, \qquad W \geq 0, \tag{3}$$

reflects the intuitively appealing notion that the marginal cost of nuclear pollution is greater the larger the stock of pollutants already in existence.

Society is also assumed to receive utility from energy output, and to consider nuclear power and fossil fuel perfect substitutes in the production of energy.

Formally, the problem facing society is to choose paths of labor, $L_1(t)$ and $L_2(t)$, to maximize the discounted value of energy output minus the disutility of pollution,

$$\int_0^\infty e^{-\delta t}\{g(L_1(t)) + f(L_2(t)) + V(W(t))\}\,dt. \tag{4}$$

The choice of $L_1(t)$ and $L_2(t)$ is subject to resource and nonnegativity constraints,

$$\bar{L}(t) \geq L_1(t) + L_2(t), \qquad L_1(t) \geq 0, \qquad L_2(t) \geq 0, \tag{5}$$

to the state equations (1) and (2), and to initial values of the state variables.

The most convenient way to analyze the solution to this problem is to form the Hamiltonian–Lagrangian function and write the corresponding necessary conditions. In current-value terms the Hamiltonian may be written

$$H = g(L_1(t)) + f(L_2(t)) + V(W(t)) - \lambda(t)g(L_1(t)) + \mu(t)\gamma f(L_2(t)) + w(t)\big(\bar{L}(t) - L_1(t) - L_2(t)\big) + q(t)L_1(t) + r(t)L_2(t), \tag{6}$$

where $\lambda(t)$ and $\mu(t)$ are the shadow prices of the stocks of fossil fuel and nuclear waste, respectively, and $w(t)$, $q(t)$, and $r(t)$ are the multipliers attached to the resource and nonnegativity constraints.[2]

The associated necessary conditions

$$\frac{\partial H}{\partial L_1} = g'(L_1)(1 - \lambda) - w + q = 0, \tag{7}$$

$$\frac{\partial H}{\partial L_2} = f'(L_2)(1 + \mu\gamma) - w + r = 0, \tag{8}$$

$$w\big(\bar{L} - L_1 - L_2\big) = 0, w \geq 0; \qquad qL_1 = 0, q \geq 0; \qquad rL_2 = 0, r \geq 0 \tag{9}$$

state that along an interior path ($q = r = 0$) labor will be employed in the production of energy until the value of the marginal product of labor equals its shadow price, $w(t)$. Since $w(t) \geq 0$, labor will be used in the production of fossil fuel (nuclear energy) only if the value of its marginal product is positive. Thus, once the price of a unit of energy ($1) exceeds the user cost of fossil fuel, production of fossil fuel will cease. Similarly, production of nuclear energy will stop once the environmental user cost of an additional kilowatt hour, $\mu\gamma$, exceeds (in absolute value) the benefit of the additional energy produced.

Along an optimal path the shadow price of fossil fuel must grow at the exogenously determined rate of discount,

$$\dot{\lambda} = \delta\lambda \Rightarrow \lambda(t) = e^{\delta t}\lambda(0), \tag{10}$$

while the shadow price of nuclear waste will change according to

$$\dot{\mu} = \delta\mu - V'(W). \tag{11}$$

The following transversality conditions must also hold [6, Theorem 1],

$$\lim_{t\to\infty} e^{-\delta t}\mu(t)W(t) = 0, \tag{12}$$

$$\lim_{t\to\infty} e^{-\delta t}\lambda(t)S(t) = 0. \tag{13}$$

It is clear from the necessary conditions that society in model I must eventually stop producing energy. From Eq. (7) fossil fuel will be produced only if λ, the user cost of the nonrenewable resource, is less than one, the marginal value of energy.

[2] The analysis below focuses on interior solutions for which $S > 0$. To treat the case of $S = 0$ one would have to add an expression $p(t)\dot{S}(t)$ to the Hamiltonian, where $p(t)$ is a multiplier which becomes positive when $S = 0$ and satisfies the condition $pS = p\dot{S} = 0$.

Equation (10), however, implies that user cost, which is growing exponentially at the rate of interest, must exceed one in finite time. Hence the stock of fossil fuel must be exhausted in finite time. It can likewise be shown that eventually production of nuclear energy will cease. From Eq. (11) if the environmental cost of nuclear energy were increasing at time t' (i.e., if μ were becoming smaller in absolute value at t') then it would increase for all $t > t'$,

$$\dot{\mu}(t') > 0 \Rightarrow \dot{\mu}(t) > 0, \qquad t \geq t'.$$

This means that μ would eventually become positive. Once $\mu > 0$, however, it would increase at a rate faster than δ, $\dot{\mu}/\mu = \delta - V'(W)/\mu > \delta$. This fact, together with $\dot{W} \geq 0$, implies that the transversality condition (12) would be violated. It thus follows that μ must decrease or remain constant for all t,

$$\dot{\mu}(t) \leq 0, \qquad \text{all } t. \tag{14}$$

Now suppose that $\dot{\mu}(t) = 0$. By (11) $\dot{W}(t)$ must also be zero and hence no nuclear energy is being produced. If, on the other hand, $\dot{\mu}(t) < 0$ then eventually $1 + \mu\gamma < 0$, implying $L_2 = 0$; or $\dot{\mu} = 0$, implying $L_2 = 0$. In either case the environmental burden imposed by nuclear pollution eventually puts an end to the production of nuclear energy.[3]

What one would like to know, of course, is how the amount of labor devoted to nuclear energy and to fossil fuel changes during the period when some energy is produced. The analysis of the model during this period is simplified by noting that the resources constraint $\bar{L} \geq L_1 + L_2$ is always binding as long as energy output is positive. In Eq. (7), for example, the fact that $g(L_1)$ is strictly increasing implies that w must be positive if $q = 0$. Since the same reasoning may be applied in (8), $w > 0$ if either $L_1 > 0$ or $L_2 > 0$. This makes it possible to substitute $\bar{L} - L_1$ for L_2 in Eq. (8) and to treat the problem as though it contained a single control variable.

During the period in which energy is produced the economy may pass through three stages:

Stage I in which all resources are devoted to the production of fossil fuels. This stage is characterized by

$$g'(\bar{L})(1 - \lambda) > f'(0)(1 + \mu\gamma); \tag{15}$$

Stage II in which both nuclear energy and fossil fuel are produced and the value of the marginal product of labor in both activities must therefore be equal,

$$g'(L_1)(1 - \lambda) = f'(\bar{L} - L_1)(1 + \mu\gamma); \tag{16}$$

Stage III during which only nuclear energy is produced and

$$g'(0)(1 - \lambda) < f'(\bar{L})(1 + \mu\gamma). \tag{17}$$

[3] To answer the question "What will society do when production of nuclear energy and fossil fuel ceases?" one could include in the integrand of (4) a flow of some inexhaustible resource, such as solar energy, which is assumed to be costlessly available to society.

The order in which the economy passes through these three stages depends on two factors, the relationship between the marginal products of labor in nuclear energy and fossil fuel production, $f'(L_2)$ and $g'(L_1)$, and the relationship between the price of fossil fuel, $\lambda(t)$, and the user cost of nuclear energy, $\mu(t)$. Intuitively, if the initial cost of producing nuclear energy were high relative to the cost of producing energy from fossil fuel one would expect society to concentrate its resources in fossil fuel, which is both cheaper and nonpolluting, switching to nuclear energy as the cost of fossil fuel rose. If, on the other hand, nuclear energy were initially cheaper to produce society would likely devote most of its resources to nuclear energy production, with fossil fuel replacing nuclear reactors as the environmental cost of nuclear pollution grew. Intuition, however, does not rule out the possibility that nuclear power might again become an attractive alternative to fossil fuel as the user cost of the latter rose.

This intuitive reasoning can be made precise by examining the phase diagram of the model in $L_1 - W$ space. To construct the diagram suppose that the economy is in Stage II ($\bar{L} > L_1 > 0$). Then the rate of change in L_1 may be obtained by differentiating (16) totally with respect to time and substituting for $\dot{\mu}$ and $\dot{\lambda}$ from (10) and (11). The end result is an equation which expresses $\dot{L}_1$ solely as a function of L_1, W and various parameters,

$$\dot{L}_1 = A\left\{\delta\left[f'(\bar{L} - L_1) - g'(L_1)\right] + V'(W)\gamma f'(\bar{L} - L_1)\right\},$$
$$A \equiv -\left[g''(L_1)(1-\lambda) + f''(\bar{L} - L_1)(1 + \mu\gamma)\right]^{-1}. \tag{18}$$

Setting $\dot{L}_1$ equal to zero and solving for W,

$$W = V'^{-1}\left\{\left[\frac{g'(L_1)}{f'(\bar{L} - L_1)} - 1\right]\frac{\delta}{\gamma}\right\}, \tag{19}$$

yields the combinations of W and L_1 which cause L_1 to remain constant along an interior path. The intercepts of (19) when $L_1 = \bar{L}$ and $L_1 = 0$ give values of W which cause the economy to remain in Stage I ($L_1 = \bar{L}$) and Stage III ($L_1 = 0$), respectively. The locus $\dot{L}_1 = 0$ is, as pictured in Figs. 1–3, positively sloped with L_1 increasing for values of W below the line and L_1 increasing above the line.

The set of points for which $\dot{W} = 0$ is given by the line $L_1 = \bar{L}$, with W increasing to the left of this line.[4]

It is clear from Figs. 1–3 that the (L_1, W) paths which are consistent with the model depend crucially on the intercepts of $\dot{L}_1 = 0$ which, in turn, depend on the relative magnitudes of $g'(0)$, $f'(\bar{L})$, $g'(\bar{L})$ and $f'(0)$. Three cases may be distinguished:

Case I. Fossil Fuel Superior: $g'(\bar{L}) > f'(0)$.

In this case fossil fuel is initially cheaper to produce than nuclear energy, in the sense that the marginal product of labor is uniformly higher in the production of fossil fuel than in the production of nuclear energy. Equations (3) and (19) together with $g'(\bar{L}) > f'(0)$ imply that the intercepts of $\dot{L}_1 = 0$ are both negative [see Fig.

[4]It should be noted that the locus $\dot{W} = 0$ is defined only in Stages I and II.

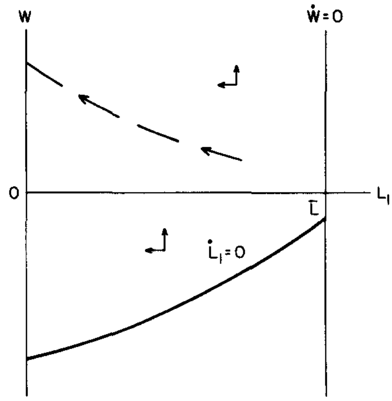

FIG. 1. Fossil Fuel Superior.

1].[5] If the optimal path is an interior path, therefore, it must resemble the path shown in Fig. 1. Along this path the fraction of labor devoted to the production of fossil fuel decreases monotonically as breeder fission replaces the nonrenewable resource. Boundary solutions, however, are also possible. For example, society may devote all of its resources to fossil fuel, the technologically more efficient, nonpolluting alternative, until fossil fuel reserves have been exhausted. All resources will then be devoted to the production of nuclear energy until pollution considerations cause nuclear energy production to cease.[6]

It is the latter policy, in fact, which turns out to be optimal. This may be proved formally by comparing welfare integrals (4) along admissible paths; however the reasoning underlying the result is quite simple. As long as society wishes to maximize the present discounted value of output net of pollution costs it will try to produce energy as fast as possible, unless there is a tradeoff between efficient production of energy and pollution creation. In Case I no such trade-off exists. Instantaneous energy output $g(L_1) + f(L_2)$ is maximized by devoting all resources to the nonpolluting alternative, L_1. Hence, over any interval during which fossil fuel reserves are positive and some resources are devoted to nuclear energy, output could be increased and the rate of increase in pollution reduced by transferring resources to fossil fuel production. It therefore follows that (4) is maximized by setting $L_1 = \bar{L}$ until $S = 0$.

Case II. Nuclear Fuel Superior: $g'(0) < f'(\bar{L})$.

Here nuclear fuel is initially a cheaper source of energy in the sense that, for any L_1, the marginal product of labor in nuclear energy exceeds the marginal product of labor in fossil fuel. This case may seem unrealistic given the present state of development of breeder reactors; however, if $g(L_1)$ is interpreted as fossil fuel output net of pollution costs, and if emission standards are stringent, the case is

[5] If $g'(\bar{L}) > f'(0)$ then the expression inside brackets in (19) is positive both when $L_1 = 0$ and when $L_1 = \bar{L}$. Since $V'(W) \leq 0$ for $W \geq 0$ (19) can be satisfied only if $W < 0$.

[6] The boundary solution $L_2 = \bar{L}$ followed by $L_1 = \bar{L}$ can be ruled out by showing that $L_1 = 0$ can never be followed by $L_1 > 0$. Note that if $L_1 = 0$, $g'(0)(1 - \lambda) < f'(\bar{L})(1 + \mu\gamma)$. In the Fossil fuel superior case this implies that $1 + \mu\gamma > 1 - \lambda$. However if this holds at t it must hold for all $t' > t$ since μ decreases more slowly than $-\lambda$.

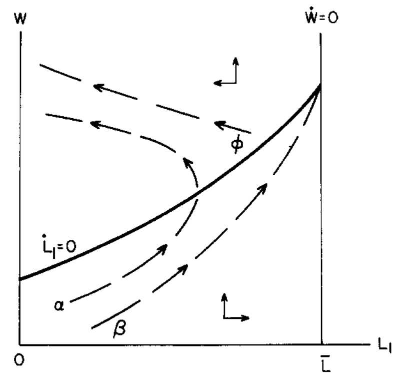

FIG. 2. Nuclear Fuel Superior.

certainly possible. When $g'(0) < f'(\bar{L})$ the intercepts of $\dot{L}_1 = 0$ are both positive [see Fig. 2] and admissible interior paths resemble the paths α, β and ϕ in Figure 2. Along paths of the form α a large portion of society's resources are initially devoted to the production of nuclear energy due to the technological superiority of breeder fission. As the stock of nuclear pollution grows, society relies more heavily on fossil fuel ($\dot{L}_1 > 0$) until finally the rising user cost of the non-renewable resources causes labor to be shifted back to the production of nuclear fuel ($\dot{L}_1 < 0$).

Paths such as β are characterized by high initial production of nuclear energy, with fossil fuel replacing nuclear energy as the environmental user cost of nuclear energy rises. Here, however, the shadow price of fossil fuel does not rise fast enough to cause $\dot{L}_1 < 0$. In the case of the path ϕ the environmental cost of nuclear waste resulting from a high $W(0)$ or high γ is great enough to outweigh the technological superiority of breeder fission and causes society to postpone the environmental costs of nuclear energy production. Hence along this path the bulk of resources are initially devoted to fossil fuel production with $\dot{L}_1 < 0$.

Which of these three paths is optimal for society depends on the severity of nuclear pollution. In the model nuclear pollution represents a more serious threat the higher is the initial stock of radionuclides and the higher is γ, the amount of

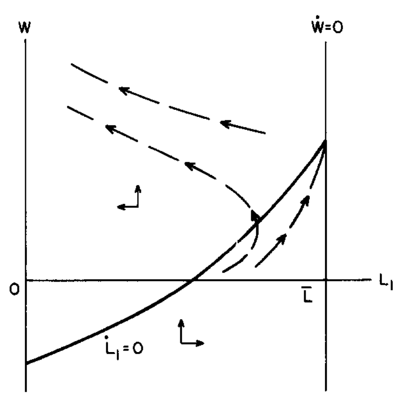

FIG. 3. The Intermediate Case.

pollution emitted for each kilowatt hour of nuclear energy produced. Intuitively, if the threat of nuclear pollution is small desires to produce output as fast as possible will cause society to devote the bulk of its resources to breeder fission. (Note that in Case II instantaneous output is maximized by setting $L_2 = \bar{L}$.) However, if pollution considerations are more important, the desire to postpone the pollution costs of nuclear energy will induce society to rely more heavily on fossil fuel. In fact, if $W(0)$ is sufficiently high,

$$W(0) > V'^{-1}\left\{\left[\frac{g'(\bar{L})}{f'(0)} - 1\right]\frac{\delta}{\gamma}\right\},$$

or, equivalently, if γ is sufficiently large, the (L_1, W) path must start above the line $\dot{L}_1 = 0$ and must be of the form ϕ. Hence if the cost of pollution is great enough, society will attempt to postpone pollution as long as possible, relying initially on fossil fuel as an energy source.

Case III. Intermediate Case: $g'(0) > f'(\bar{L}), f'(0) > g'(\bar{L})$.

If neither energy source is initially cheaper than the other for all levels of output then the $\dot{L}_1 = 0$ locus appears as pictured in Fig. 3, and the possible paths of fossil fuel production are qualitatively similar to those in Case II.

3. MODEL II—NUCLEAR POWER INEXHAUSTIBLE

We now take a more optimistic view of nuclear technology and assume that if sufficient resources are devoted to nuclear safety all residuals created by the breeder reactor can be contained. In practice the main sources of solid waste emissions are likely to be accidents which occur when spent fuel is transported to and from reprocessing plants, and leakages of solid waste from disposal sites. The frequency and size of transportation accidents can be reduced by allowing spent fuel to cool down for longer periods before being shipped to reprocessing plants and by transporting fissionable material in small packages surrounded by large quantities of inert material. Leakages of solid waste from burial sites can likewise be reduced by devoting more resources to the safe packaging of wastes and by frequently monitoring burial sites. The question open to debate is whether the amount of radioactive material released into the environment can be reduced to zero (or to an insignificant amount) by such safety measures. In model II we assume that it can, hence production of nuclear energy need not increase the stock of plutonium in the environment.

Formally, let L_3 be the amount of labor devoted to nuclear pollution control. If E_2 is the amount of nuclear energy produced then the stock of nuclear waste should increase by some function $G(L_3, E_2)$, where G is decreasing in L_3 and increasing in E_2. To keep matters simple we assume that G is additively separable in L_3 and E_2:

$$\dot{W} = \gamma E_2 - h(L_3). \tag{20}$$

In (20) γE_2 should be interpreted as the amount of radioactive material which would be released into the environment if no resources were spent on pollution control. $h(L_3)$ represents the amount of pollution safely contained, i.e., the amount of spent fuel safely recycled or buried as waste,

$$h(0) = 0, \qquad h'(L_3) > 0, \qquad h''(L_3) < 0, \qquad h'(0) < \infty.$$

Since plutonium once released into the environment cannot easily be recaptured, W must be nondecreasing, implying that

$$\gamma E_2 \geq h(L_3). \tag{21}$$

With the addition of L_3, our model now contains two independent decision variables, assuming that the resource constraint $\bar{L} \geq L_1 + L_2 + L_3$ is binding. Since control problems with two state and two decision variables are usually difficult to handle we shall simplify matters by assuming that nuclear energy is produced under constant returns to scale,

$$E_2 = \zeta L_2, \qquad \zeta > 0. \tag{22}$$

The other assumptions of model I are assumed to hold unchanged.

The problem of choosing between nuclear fuel and fossil fuel as energy sources is now one of picking paths $L_1(t)$, $L_2(t)$ and $L_3(t)$ to maximize

$$\int_0^{\infty} e^{-\delta t}\{g(L_1(t)) + \zeta L_2(t) + V(W(T))\}\, dt \tag{23}$$

subject to the transition equations

$$\dot{W}(t) = \gamma\zeta L_2(t) - h(L_3(t)), \tag{24}$$

$$\dot{S}(t) = -g(L_1(t)) \tag{25}$$

and to various constraints on the control variables

$$\bar{L}(t) \geq L_1(t) + L_2(t) + L_3(t), \tag{26}$$

$$\gamma\zeta L_2(t) - h(L_3(t)) \geq 0, \tag{27}$$

$$L_1(t) \geq 0, \qquad L_2(t) \geq 0, \qquad L_3(t) \geq 0. \tag{28}$$

Before stating the necessary conditions corresponding to this problem we note that the resource constraint (26) will always be binding along an optimal path: Society can always improve its welfare by allocating unused resources to the production of nuclear energy in such a way that the stock of nuclear waste remains unchanged. It is therefore possible to substitute $\bar{L} - L_1 - L_2$ for L_3 and treat the problem as a two-control-variable problem. With this modification, the current-value Hamiltonian–Lagrangian function and necessary conditions are given in

(29)–(36).

$$H = g(L_1) + \zeta L_2 + V(W) - \lambda g(L_1) + \mu\left[\gamma\zeta L_2 - h(\bar{L} - L_1 - L_2)\right]$$
$$+ p\left[\gamma\zeta L_2 - h(\bar{L} - L_1 - L_2)\right] + qL_1 + rL_2 + w(\bar{L} - L_1 - L_2), \quad (29)$$

$$\frac{\partial H}{\partial L_1} = g'(L_1)(1 - \lambda) + \mu h'(\bar{L} - L_1 - L_2)$$
$$+ q - w + ph'(\bar{L} - L_1 - L_2) = 0, \quad (30)$$

$$\frac{\partial H}{\partial L_2} = \zeta(1 + \mu\gamma) + \mu h'(\bar{L} - L_1 - L_2) + r - w + p\left[\gamma\zeta + h'(\bar{L} - L_1 - L_2)\right]$$
$$= 0, \quad (31)$$

$$\begin{aligned} &p\left[\gamma\zeta L_2 - h(\bar{L} - L_1 - L_2)\right] = 0, && p \geq 0, \\ &qL_1 = 0, \quad q \geq 0; \quad rL_2 = 0, && r \geq 0, \\ &w(\bar{L} - L_1 - L_2) = 0, && w \geq 0, \end{aligned} \quad (32)$$

$$\dot{\lambda} = \delta\lambda, \quad (33)$$

$$\dot{\mu} = \delta\mu - V'(W), \quad (34)$$

$$\lim_{t\to\infty} e^{-\delta t}\mu(t)W(t) = 0, \quad (35)$$

$$\lim_{t\to\infty} e^{-\delta t}\lambda(t)S(t) = 0. \quad (36)$$

In these equations, the value of allocating an additional hour of labor to nuclear safety is the product of the utility gained by preventing W from increasing, $-\mu$, times the amount by which W does not increase, $h'(\bar{L} - L_1 - L_2)$. Equations (30) and (31) thus state that along an interior path ($p = q = r = w = 0$) the value of the marginal product of labor must be equal in all three activities—production of fossil fuel, production of nuclear energy, and nuclear pollution control. Otherwise the opportunity cost of fossil fuel production is either the marginal value of producing nuclear energy or the marginal value of pollution control, whichever is greater.

In model I society devoted more of its resources to the production of fossil fuel the larger the environmental cost of nuclear energy (the smaller μ) relative to the user cost of fossil fuel. Now a large absolute value of μ, relative to λ, need not imply a large value of L_1 since it also implies that the marginal benefits of pollution containment are high.

In addition to the interior solution (Stage IV), Eqs. (30)–(32) imply that four other stages of energy production are possible. Stage I, in which all resources are devoted to the production of fossil fuel ($L_1 = \bar{L}$, $L_2 = L_3 = 0$); Stage II, in which all labor is allocated to the production of breeder fission ($L_2 = \bar{L}$, $L_1 = L_3 = 0$), Stage III, in which fossil fuel and nuclear energy are both produced but no resources are devoted to nuclear pollution control ($L_1 > 0, L_2 > 0, L_3 = 0$), and

Stage V, in which labor is divided between the production of nuclear energy and pollution control ($L_1 = 0, L_2 > 0, L_3 > 0$).

The order in which the economy passes through these stages and the behavior of output within each stage depends, as in model I, on whether fossil fuel or nuclear energy is initially cheaper to produce. Here we shall analyze the two polar cases of Section 2—Fossil Fuel Superior and Nuclear Fuel Superior. It is convenient to begin our analysis by deriving equations for the rates of change in L_1, L_2, and L_3 along an interior path. If we assume that the constraint $\dot{W} \geq 0$ is not binding, then the necessary conditions reduce to

$$g'(L_1)(1-\lambda) + \mu h'\left(\bar{L} - L_1 - L_2\right) = 0, \tag{30$'$}$$

$$\zeta(1+\mu\gamma) + \mu h'\left(\bar{L} - L_1 - L_2\right) = 0. \tag{31$'$}$$

These may be differentiated totally with respect to time to obtain two equations in $\dot{L}_1$ and $\dot{L}_2$,

$$B\dot{L}_1 - \dot{\lambda} g' + \dot{\mu} h' - \mu h'' \dot{L}_2 = 0, \qquad B \equiv (1-\lambda)g'' - \mu h'', \tag{37}$$

$$\dot{\mu}(\zeta\gamma + h') - \mu h'' \dot{L}_1 - \mu h'' \dot{L}_2 = 0 \tag{38}$$

Subtracting (38) from (37) and substituting for $\dot{\lambda}$ and $\dot{\mu}$ from (33) and (34) yields an equation for the rate of change in L_1,

$$\dot{L}_1 = \frac{-1}{(1-\lambda)g''}\{-\delta\lambda g' - \delta\mu\lambda\zeta + V'(W)\gamma\zeta\}. \tag{39}$$

After some rewriting[7] the rate of change in L_1 is seen to be identical to $\dot{L}_1$ model I (cf. (18)) under the assumption that $f'(L_2) = \zeta$,

$$\dot{L}_1 = -\frac{1}{(1-\lambda)g''}\{-\delta[g'(L_1) - \zeta] + V'(W)\zeta\gamma\}. \tag{40}$$

This should not be surprising. In model I fossil fuel output is determined along an interior path by equating the value of the marginal product of labor in fossil fuel production to the value of the marginal product of labor in nuclear energy production. Here, the value of the marginal product of labor in pollution control must also be considered; however, since it must equal the value of the marginal product of labor in nuclear energy (along an interior path) its inclusion does not affect the behavior of L_1.

The behavior of nuclear energy production is, however, affected by opportunities for pollution control. From (38) the sign of $\dot{L}_2$ depends on the rate of change in μ, the shadow price of nuclear waste, as well as on the rate of change in L_1. Specifically, an increase in the user cost of nuclear energy (a decrease in μ) decreases nuclear energy output since it implies that the benefits of pollution abatement are rising. For the same reason, a decrease in μ increases expenditure on

[7]Note that (30)′ and (31)′ imply that $\mu = [(1-\lambda)g' - \zeta]/\gamma\zeta$ along an interior path. If this is substituted into (39), (40) is obtained.

nuclear safety,

$$\dot{L}_3 = -\dot{L}_1 - \dot{L}_2 = -\dot{\mu}\frac{(\zeta\gamma + h')}{\mu h''}. \tag{41}$$

Energy Production in the Fossil Fuel Superior Case, $g'(\bar{L}) > \zeta$

When fossil fuel, the nonpolluting energy source, is also cheaper to produce than nuclear energy society will first rely entirely on fossil fuel switching to nuclear energy only when fossil fuel reserves have been exhausted. The reasoning used to justify this policy in Section 2 is in no way affected by opportunities for pollution cleanup. By devoting all resources to fossil fuel society can produce energy as fast as possible without incurring pollution costs. Even if nuclear safety devices were extremely efficient in containing pollution, the best that could be achieved by devoting resources to pollution control would be to render nuclear energy a pollution-free energy source. However, as long as $g'(\bar{L}) > \zeta$, nuclear energy would still not be as productive as fossil fuel, and the present discounted value of output could be increased by devoting resources to fossil fuel production.

Once fossil fuel reserves are exhausted society will enter the Nuclear Age (Stages IV–V). Initially all resources may be devoted to the production of nuclear energy and none to pollution containment,

$$\zeta(1 + \mu\gamma) > \mu h'(0). \tag{42}$$

However it may be shown, using an argument similar to that above, that the environmental user cost of nuclear energy will rise over time ($\dot{\mu} \leq 0$).[8] Equation (42) will therefore eventually be violated and some resources will be devoted to pollution control. The argument which guarantees that $\dot{\mu} \leq 0$ also guarantees by (38) and (40) that nuclear energy production falls during Stage V while expenditure on nuclear safety rises. This is what one would expect: if the environmental cost of nuclear energy production increases over time while the marginal benefits of energy production remain constant, production of nuclear energy should decrease.

Once in Stage V the economy eventually arrives as a stationary state in which as much nuclear energy is produced as is possible without increasing the stock of nuclear pollution. The fact that the pollution stock remains constant in the steady state is sufficient to determine the stationary value of L_2, L_2^∞.

$$\gamma\zeta L_2^\infty = h(\bar{L} - L_2^\infty). \tag{43}$$

Clearly nuclear energy production will be larger the smaller is γ, the amount of pollution which accompanies each kilowatt hour of nuclear energy.

[8] As in model I, μ cannot increase along an optimal path by virtue of (34) and the transversality condition (35). Equation (34) implies that if ever μ begins to increase it will continue to increase thereafter. This implies that μ will eventually grow at a rate greater than δ and that the transversality condition will be violated.

The steady state values of the stock of nuclear pollution, W^{∞}, and of its shadow price, μ^{∞}, are jointly determined by

$$\zeta(1 + \mu^{\infty}\gamma) + \mu^{\infty}h'(\bar{L} - L_2^{\infty}) = 0, \tag{31$'$}$$

and the condition $\dot{\mu}^{\infty} = 0$,

$$\mu^{\infty} = \frac{V'(W^{\infty})}{\delta}. \tag{44}$$

The latter condition states that cost of increasing the pollution stock by one more unit is the discounted value of the stream of disutility, $V'(W)$, caused by a marginal increase in W^{∞}. As intuition suggests, the steady-state stock of nuclear waste will be greater the higher society's rate of discount, since the higher δ the lower the value placed on the costs of nuclear pollution borne by future generations. On the other hand, increasing γ, the marginal pollution costs of breeder fission, has an ambiguous effect on W^{∞}.[9]

Energy Production When Nuclear Energy is Technically Superior, $\zeta > g'(0)$

In the Fossil Fuel Superior case opportunities for nuclear pollution control do not affect the rate at which society uses fossil fuel reserves. Reserves are depleted as rapidly as possible, as in model I. Society then enters a Nuclear Age in which energy production and expenditure on pollution control behave as they would in a standard model of waste accumulation and disposal [4, 7]. The Nuclear Fuel Superior case is more interesting, for here the possibility of pollution cleanup may qualitatively alter the path of fossil fuel production. To see why this may occur suppose that nuclear safety devices are so efficient that one unit of labor allocated to pollution control cleans up as much pollution as society is capable of generating at any instant, $\gamma\zeta\bar{L}$. In this case nuclear energy would be a pollution-free, technologically superior alternative to fossil fuel and society would never use its reserves of fossil fuel at all. The same result will of course obtain under less extreme circumstances, with the exact conditions for the nonuse of fossil fuel depending on the relationship between $g(L_1)$, ζL_2 and $h(L_3)$.

We shall now derive conditions sufficient to guarantee that fossil fuel reserves will never be mined. For this to occur it is sufficient that the value of the marginal product of labor in nuclear energy (net of pollution costs) exceed the marginal product of labor in fossil fuel,

$$\zeta(1 + \mu\gamma) > g'(0), \qquad \text{for all } t. \tag{45}$$

However, because $\mu(t) \geq V'(W^{\infty})/\delta$, (45) will be satisfied provided

$$\zeta\left(1 + \frac{V'(W^{\infty})\gamma}{\delta}\right) > g'(0). \tag{46}$$

[9] These results may be verified by applying the implicit function theorem to the set of equations (31)$'$, (43), and (44).

In order to express this condition in terms of the marginal products of labor we may substitute equation (44) into (31)′ to obtain

$$V'(W^{\infty}) = \frac{-\zeta\delta}{\gamma\zeta + h'(\bar{L} - L_2^{\infty})}. \tag{47}$$

Substituting this into (46) and rearranging terms yields as the sufficient condition

$$\frac{\zeta h'(\bar{L} - L_2^{\infty})}{\gamma\zeta + h'(\bar{L} - L_2^{\infty})} > g'(0), \tag{48}$$

where L_2^{∞}, the steady state value of nuclear energy, is determined by (43).

Note that the left-hand side of (48) is increasing in h'. Thus, as one would expect, fossil fuel reserves will be left idle if the marginal product of labor in pollution control is high relative to the marginal product of labor in fossil fuel production.

If Eq. (48) is satisfied and no fossil fuel is produced the economy will proceed through Stages II and V with nuclear energy production declining through time and expenditure on nuclear safety increasing as society approaches the stationary state. If, on the other hand, fossil fuel production is competitive with nuclear energy the economy will pass through Stage IV ($L_1 > 0, L_2 > 0, L_3 > 0$), possibly preceded by Stage III, before entering the Nuclear Age. What we are interested in knowing is the shape of the fossil fuel (L_1) and nuclear energy (L_2) paths during this "Fossil Fuel Era." Specifically, how does the opportunity for allocating resources to pollution control affect the production of fossil fuel? Furthermore, how should resources devoted to breeder fission be divided between energy production and expenditure on pollution control?

The behavior of fossil fuel (L_1) during Stages III and IV can be analyzed using a phase diagram similar to Fig. 2. Since the equation for the rate of change in L_1 (40) is the same as in model I, the $\dot{L}_1 = 0$ locus pictured in Fig. 4 is identical to that in Fig. 2. What is different is the set of points for which $\dot{W} = 0$. In model II the rate of change in W depends on L_3 as well as on L_1,

$$\dot{W} = \gamma\zeta(\bar{L} - L_1 - L_3) - h(L_3). \tag{49}$$

Hence the $\dot{W} = 0$ locus, which has the equation

$$L_1 = \bar{L} - L_3 - \frac{h(L_3)}{\gamma\zeta}, \tag{50}$$

must be drawn conditional on L_3. In Fig. 4, then, this locus is pictured as a vertical line which moves leftward over time as L_3 increases (see (41)).

In examining possible L_1 paths it should be remembered that any admissible path must remain to the left of the current $\dot{W} = 0$ locus. This implies that paths of the form β can never go beyond the "leftmost" $\dot{W} = 0$ locus, for if they did they would eventually violate the $\dot{W} \geq 0$ constraint. Paths of the form α and ϕ are both admissible as long as they move ahead of the line $\dot{W} = 0$. Figure 4 indicates, then, that the behavior of fossil fuel output in Stage IV is similar to that in model I (cf.

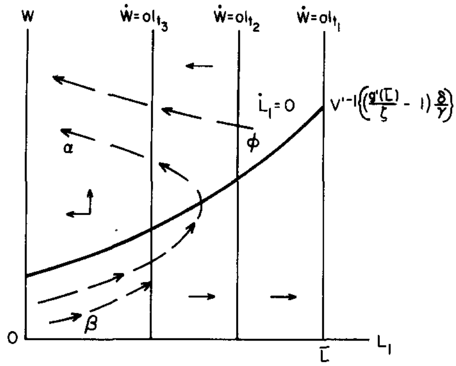

FIG. 4. Nuclear Fuel Superior, pollution control feasible.

Fig. 2) with the precise shape of the $L_1(t)$ path depending on the severity of nuclear pollution. Thus opportunities for pollution control do not qualitatively alter the behavior of fossil fuel output, provided fossil fuel is produced at all.[10] The path of nuclear energy production may, however, differ qualitatively from that in model I.

Consider, for example, the case in which pollution considerations cause society to rely initially on fossil fuel in an effort to postpone the costs of nuclear pollution. This will occur if the initial pollution stock is high,

$$W(0) > V'^{-1}\left\{\left(\frac{g'(\bar{L})}{\zeta} - 1\right)\frac{\delta}{\gamma}\right\},$$

In this case, the optimal L_1 path resembles ϕ in Fig. 4. If there were no opportunities for pollution control a decrease in fossil fuel production over time would necessarily imply an increase in nuclear energy production. When resources may be allocated to nuclear safety this is no longer the case. As Eq. (38) indicates, the increasing shadow price of nuclear waste ($\dot{\mu} < 0$) may cause nuclear energy output to decrease over time. Alternately, production of nuclear fuel may initially increase, reach a peak, and then decrease as resources are transferred from the production of energy to expenditure on pollution control.[11] Eventually, of course, fossil fuel reserves will be exhausted and society will enter Stage V, during which nuclear energy output declines steadily as the economy approaches the stationary state.

4. CONCLUSION

In the models developed above an economy with fixed resources for energy production must decide what portion of its energy needs will be met by fossil fuel and what portion by nuclear energy. The outcome of this decision depends, of course, on the cost of fossil fuel production versus the cost of nuclear energy;

[10] If the $\dot{W} = 0$ line were to reach the W-axis during Stage IV paths of the form β would be ruled out altogether since they would violate the constraint $W \geq 0$.

[11] The L_2 paths described here are only two of many admissible paths. Unfortunately, one cannot even guarantee that the L_2 path is unimodal.

however, as is emphasized above, the pollution effects of each alternative are equally important in determining the pattern of energy use. Of the two pollution problems the paper focuses on pollution associated with breeder fission, both because of the serious health effects of radiation and because of the long-lived nature of radioactive isotopes.

Throughout the paper society's objective is to maximize the present discounted value of energy output net of pollution costs. Thus intuitively the optimal policy is to produce output as fast as possible by devoting all resources to the more efficient energy source, unless the more efficient energy source entails higher pollution costs. In the case in which fossil fuel, the nonpolluting alternative, is also a more efficient source of fuel than nuclear energy society initially devotes all its resources to fossil fuel production. Only when fossil fuel reserves have been exhausted is breeder fission used as an energy source. This result continues to hold even if nuclear safety devices make it possible cheaply to contain radioactive pollution: As long as nuclear energy remains technologically inferior, society maximizes the present discounted value of output by producing fossil fuel first.

The optimal energy policy is more complicated in the case in which nuclear energy is cheaper to produce than fossil fuel. In this case society must balance desires to maximize output against desires to postpone the pollution costs of breeder fission. Which motive is stronger depends on the severity of the nuclear pollution problem, i.e., on the size of the initial pollution stock and on the amount of pollution which accompanies each kilowatt hour of nuclear energy production. If the initial stock of nuclear pollution is sufficiently high then desires to postpone pollution predominate and society will at first devote the bulk of its resources to fossil fuel production with fossil fuel output declining over time. If on the other hand the pollution stock is initially low, implying that the marginal environmental cost of nuclear energy is low, resources will be concentrated on the production of nuclear energy with fossil fuel replacing nuclear energy as the environmental cost of nuclear energy rises. In either case nuclear energy production, like fossil fuel production, must eventually cease as long as there is some irreducible amount of pollution associated with nuclear energy.

These results are altered in several ways once opportunities for pollution control are introduced. For example, when nuclear energy is an inexhaustible energy source it is possible that fossil fuel reserves will never be used at all. This will occur if the marginal product of labor in pollution abatement is high relative to the marginal product of labor in fossil fuel, implying that nuclear energy is a low-pollution, low-cost energy source. If fossil fuel is produced then the path of fossil fuel output will depend, as above, on the size of the initial pollution stock and will have the general shape described in the preceding paragraph. The path of nuclear energy production, however, will be influenced by opportunities for pollution control. Indeed, in the important case in which the initial waste stock is high and fossil fuel production optimally decreases over time, nuclear energy production may decrease rather than increase along an interior path or may exhibit a hump-shaped pattern with society gradually increasing its output of nuclear energy. After nuclear energy output reaches a peak energy production begins to decline as resources are transferred to pollution control.

Once fossil fuel reserves have been exhausted the trade-offs which society faces are those of a standard model of waste creation and disposal. The difference here is that because of the dispersed nature of radioactive waste no net pollution cleanup is possible. Production of nuclear energy decreases steadily over time and expenditure on nuclear safety increases as the environmental user cost of nuclear energy rises. Eventually, a steady state is reached in which expenditure on nuclear safety is just sufficient to assure that nuclear energy production, on net, creates no pollution.

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