THE OPTIMAL CONSUMPTION OF DEPLETABLE NATURAL RESOURCES: AN ELABORATION, CORRECTION, AND EXTENSION*

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Competitive behavior in an extractive industry with increasing marginal costs of production will generate a socially efficient pattern of production over time. The Weinstein-Zeckhauser[1] analysis of this result assumed that marginal extraction costs at a mine depend only on the amount extracted from the mine to date. This note clarifies the original Weinstein-Zeckhauser proof and goes on to consider more explicitly what happens when extraction costs depend not only on the amount extracted at each site, but also on the firm doing the extraction. When costs of extraction at a site depend only on the cumulative amount extracted there to date, all that is required for efficiency is that there be competitive markets for the extracted commodity. Section II of this note shows that the optimality result need not apply if technological capabilities differ among firms (or nations), so that the cost of extraction depends as well on the firm extracting the resource. However, if there exist competitive markets for trading extraction technologies or extraction rights, an efficient outcome is guaranteed. This situation of differential extraction costs is obviously of considerable policy relevance, particularly as the ownership of natural resource deposits shifts increasingly toward the developing world and away from the more technologically advanced nations. In recognition of this situation, both categories of nations have made recent proposals to facilitate the trade and transfer of extractive technologies.

Section I of this note elaborates and corrects the discussion of the case of increasing marginal extraction cost in the Weinstein-Zeckhauser analysis. That discussion did not consider the important particular case where some or all producers run up against a constraint on their total resources, where corner solutions may prevail. The

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substantive assertions in that analysis are valid, though their proof that the competitive equilibrium is socially optimal was incomplete.

I. Optimality of Competitive Markets with Increasing Marginal Extraction Costs—A Correction and Elaboration

Three aspects of the original presentation of this case (pages 379–81) might lead to confusion. First, the symbols q_t and q_t^k were inappropriately used to denote both the quantity supplied in year t, and the cumulative quantity supplied up through year t. They should apply to the former. Cumulative quantities should be denoted by x_t and x_t^k, where

$$x_t = \sum_{i=0}^{t} q_i$$

and

$$x_t^k = \sum_{i=0}^{t} q_i^k.$$

To correct this, the letter q should be replaced by x throughout the first paragraph of page 379, and in the second and third lines of the last paragraph of page 380. In addition, equation (13) should be followed by, "where q_i^k is the quantity supplied by individual producer k in period i."

Second, the subscripts in equations (12) and (13) are incorrect, and the possibility of corner solutions is ignored. The correct conditions are as follows:

$$(12)\quad d_t(q_t^*) - c'\left(\sum_{i=0}^{t} q_i^*\right) - \sum_{j=t+1}^{\infty} (1+r)^{t-j} \times \left[c'\left(\sum_{i=0}^{j} q_i^*\right) - c'\left(\sum_{i=0}^{j-1} q_i^*\right)\right] \leq (1+r)^t \lambda,$$

where equality holds if $q_t > 0$, and

$$(13)\quad p_t - c_k'\left(\sum_{i=0}^{t} q_i^k\right) - \sum_{j=t+1}^{\infty} (1+r)^{t-j} \times \left[c_k'\left(\sum_{i=0}^{j} q_i^k\right) - c_k'\left(\sum_{i=0}^{j-1} q_i^k\right)\right] \leq (1+r)^t \lambda^k,$$

where equality holds if $q_t^k > 0$. In the remainder of this discussion, equations (12) and (13) refer to these corrected conditions.

Third, we did not show that, in general, the shadow prices (the λ^k's) will be the same for all suppliers. If they were not equal, equation (13) would not imply that the optimality equation (12) must hold in competitive equilibrium. A few steps are needed to show that for all k, $\lambda^k = \lambda$, and that the stated result is correct. A complete proof follows, superseding the analysis given on the top of page 381.

To demonstrate that the equilibrium conditions (13) imply the optimality conditions (12) and (12a) requires several steps. First, it is shown that each producer k who is supplying a nonzero quantity in period t must be operating at the same marginal cost of extraction (i.e., $c_k'(\Sigma_{i=0}^t q_i^k)$ must be equal for all k). Next, it is shown that the shadow prices are equal for all suppliers ($\lambda^k = \lambda$, all k). Finally, the proof that (13) implies (12) is completed, allowing for the possibility that some producers may be supplying zero resource in some periods and, therefore, that (13) holds as an inequality for those periods.

First, note that (13) in period $(t+1)$ is written as

$$\text{(i)}\quad p_{t+1} - c_k'\left(\sum_{i=0}^{t+1} q_i^k\right) - \sum_{j=t+2}^{\infty} (1+r)^{t+1-j} \times \left[c_k'\left(\sum_{i=0}^{j} q_i^k\right) - c_k'\left(\sum_{i=0}^{j-1} q_i^k\right)\right] = (1+r)^{t+1}\lambda^k,$$

assuming that $q_{t+1}^k > 0$. Note also that (13) in period (t) may be rewritten as

$$\text{(ii)}\quad p_t - c_k'\left(\sum_{i=0}^{t} q_i^k\right) - (1+r)^{-1}\left[c_k'\left(\sum_{i=0}^{t+1} q_i^k\right) - c_k'\left(\sum_{i=0}^{t} q_i^k\right)\right] - \sum_{j=t+2}^{\infty} (1+r)^{t-j}\left[c_k'\left(\sum_{i=0}^{j} q_i^k\right) - c_k'\left(\sum_{i=0}^{j-1} q_i^k\right)\right] = (1+r)^{t}\lambda^k,$$

by extracting the $(t+1)$ term in the summation. Multiplying (ii) through by $(1+r)$ and subtracting (ii) from (i) yields

$$\text{(iii)}\quad \left[p_{t+1} - c_k'\left(\sum_{i=0}^{t} q_i^k\right)\right] - (1+r)\left[p_t - c_k'\left(\sum_{i=0}^{t} q_i^k\right)\right] = 0.$$

Since the sequence of $\{p_t\}$ are taken as constants by the suppliers, equation (iii) determines

$$c_k'\left(\sum_{i=0}^{t} q_i^k\right)$$

uniquely for all suppliers. This proves the first assertion, that all suppliers of a nonzero quantity of the resource must be operating at the same marginal cost of extraction.

Next, since the marginal costs are identical for all suppliers, the aggregate (industry) marginal cost is uniquely defined as the horizontal sum of the individual marginal costs, so that for any k for which $q_i^k > 0$,

$$\text{(iv)}\; c_k'\left(\sum_{i=0}^{t} q_i^k\right) = c'\left(\sum_k \sum_{i=0}^{t} q_i^k\right) = c'\left(\sum_{i=0}^{t} \sum_k q_i^k\right) = c'\left(\sum_{i=0}^{t} q_i\right),$$

where $q_i = \Sigma_k q_i^k$ by definition. To demonstrate that $\lambda^k = \lambda$ for all k, let t^* be any period for which $q_{t^*}^k > 0$. Then (13) holds with equality for $t = t^*$. Substituting (iv) into the left-hand side of (13) and using the fact that $p_t = d_t(q_t)$ in the equilibrium yields

$$(13')\quad d_t(q_t) - c'\left(\sum_{i=0}^{t} q_i\right) - \sum_{j=t+1}^{\infty} (1+r)^{t-j} \times \left[c'\left(\sum_{i=0}^{j} q_i\right) - c'\left(\sum_{i=0}^{j-1} q_i\right)\right] = (1+r)^t \lambda^k.$$

Since (13′) holds for all t for which $q_t > 0$, this set of equations, together with the resource constraint

$$\sum_{t=0}^{\infty} q_t = Q,$$

may be solved to obtain the equilibrium $\{q_t^k\}$ and the value of λ^k. This procedure may be repeated for (12) to yield the optimal $\{q_t^*\}$ and λ. Since equations (12) and (13′) are structurally identical, it follows that $\lambda = \lambda^k$.

It is an easy step, given that $\lambda = \lambda^k$, to see that if (13) holds with equality for all k and t, then (12) also holds. Simply use (iv) to substitute c' for c_k' and q_i for q_i^k where they appear in (13). It can then be concluded that the market equilibrium (13) reduces to the optimizing conditions (12), and the allocation is seen to be efficient.

In the case where $q_t^k = 0$ for a particular time t and supplier k (as would occur, for example, if the supplier had already exhausted his fixed stock), condition (iv) would no longer hold for supplier k in period t. However, the aggregate marginal cost c' can now be defined more accurately as the horizontal sum of the marginal costs for those suppliers *who are producing* in period t. Since the last two equalities in (iv) continue to hold, the fact that (13) holds with equality for all k with $q_i^k > 0$ is sufficient to guarantee (12). If for some t *all* $q_t^k = 0$, then (13) would be an inequality for all k, implying that (12) would also be an inequality and, therefore, that zero production in period

t is optimal. This completes the demonstration of optimality for the competitive equilibrium.

II. Cases Where Extraction Rights or Extraction Technologies Must Be Traded

When costs of extraction depend on the cumulative amount extracted at each site, regardless of who extracts the resource, all that is required for efficiency is that there be competitive markets for the extracted commodity. Additional markets are required for efficiency, however, when technologies differ among firms (or nations), so that the cost of extraction depends on who is extracting the resource. The only structural difference between this and the preceding case is that there are now several cost functions associated with each deposit (one for each firm) instead of just one.

When all firms have identical technologies, there is no incentive or efficiency need for firms to trade initial endowments of the depletable resource. In the case we now consider, where technologies differ, it will generally be profitable for firms to trade resource holdings (or their short-term equivalent, extraction rights) or alternatively to trade extractive technologies (or their short-term equivalent, extraction services). Both types of trades take place frequently in the real world, especially between developing countries that lack the technological capability to develop their resource holdings in the most economical fashion, and the more technologically advanced nations. Moreover, the addition of markets in which such trades can be conducted will guarantee that patterns of extraction determined by competitive processes will be efficient. Before showing this, we shall first present a negative result.

If there are no markets to trade technologies or extraction rights, and if technologies differ among firms, patterns of extraction will not in general be globally optimal. To demonstrate this, suppose that there are K deposits of a nonrenewable resource, each of size Q^k and each owned by a different firm (country). Let $c_{hk}(x^k)$ denote the cost to firm h of extracting the first x^k units at site k regardless of when extraction takes place or who has done the previous extraction. The cost of extracting q_t^{hk} units at time t is thus given by

$$c_{hk}(x_{t-1}^k + q_t^{hk}) - c_{hk}(x_{t-1}^k),$$

where

$$x_t^k \equiv \sum_h \sum_{i=0}^{t} q_i^{hk}$$

is the total amount extracted by all firms at site k.[2]

Given demand functions $d_t(q_t)$, the socially efficient rate of extraction for each firm at each site is determined by selecting each q_t^{hk} to maximize

$$\sum_{t=0}^{\infty} (1+r)^{-t} \left\{ \int_0^{q_t} d_t(\xi_t) d\xi_t - \sum_h \sum_k [c_{hk}(x_{t-1}^k + q_t^{hk}) - c_{hk}(x_{t-1}^k)] \right\}$$

subject to

$$\sum_h \sum_t q_t^{hk} \leq Q^k, \quad k = 1, 2, \ldots, K,$$

and

$$q_t = \sum_h \sum_k q_t^{hk}.$$

If λ^k is the multiplier attached to the kth resource constraint (and $\lambda^k = \lambda$ for all k by the argument given above), then optimal output by firm h at site k must satisfy

$$\text{(v)} \quad d_t(q_t) - c'_{hk}(x_{t-1}^k + q_t^{hk}) - \sum_h \sum_{j=t+1}^{\infty} (1+r)^{t-j} [c'_{hk}(x_{j-1}^k + q_j^{hk}) - c'_{hk}(x_{j-1}^k)] \leq (1+r)^t \lambda^k \qquad k = 1, 2, \ldots, K; \quad h = 1, 2, \ldots, H,$$

where equality holds if $q_t^{hk} > 0$. Equation (v) says that at site k and time t each firm should produce until price minus discounted marginal cost equals $(1+r)^t \lambda^k$, the user cost of the resource at that site. Note that the marginal cost of production by firm h includes the increase in future extraction costs for all firms at the site.

The paths of output satisfying (v) may be contrasted with the competitive rates of extraction determined in a sequence of spot markets by firms who own their own mines. When no opportunities for trade exist, each firm, faced with a path of prices $\{p_t\}$ will choose a path of extraction $\{q_t^{kk}\}$ to maximize

$$\sum_{t=0}^{\infty} (1+r)^{-t} [p_t q_t^{kk} - c_{kk}(x_{t-1}^k + q_t^{kk}) + c_{kk}(x_{t-1}^k)]$$

subject to

$$\sum_t q_t^{kk} \leq Q^k.$$

The competitive paths of extraction will not, in general, be socially efficient, since they imply that $q_t^{hk} = 0$ for $h \neq k$.

As long as certain firms have a comparative advantage in mining the resource deposits of other firms, however, incentives for trade, either in initial endowments or in extractive services, exist. If each resource owner can purchase extractive services from other firms in the industry, and if markets for extractive services are competitive—that is, if there are many firms relative to methods of extraction—then each mine owner effectively faces a marginal cost of extraction curve given by

$$c'_k(x^k) = \min_h \, [c'_{hk}(x^k)]$$

and will determine the efficient amount to be extracted by all firms at his deposit. However, this is the same problem faced by a planner who wishes to determine the efficient rate of extraction for each firm at each site, and the competitive and efficient paths of extraction must therefore coincide.

Equivalently, if owners of technologies can purchase (or lease) the right to extract from a given resource deposit, the result will be globally optimal. This is because the asset value of the deposit (or rental value of some portion of the deposit) will reflect the maximum expected discounted profit among firms in the market. This maximum will be characterized by minimum marginal extraction cost among firms. Of course, the asset value to this high-bidding firm will also reflect its opportunity to resell the deposit at a later date to another firm that might be more efficient along another portion of the extraction cost curve (for example, deeper drilling technology as opposed to surface extraction technology).

III. Conclusion

Any model of an extractive industry may overlook or underplay real world complications. We should therefore be cautious when extrapolating from theoretical demonstrations of efficiency to the conclusion that particular configurations of markets will automatically generate efficient outcomes. So long as there is no fundamental violation of the conditions for a competitive equilibrium in the extraction processes themselves, a sufficiently rich variety of available markets (such as those to trade spot commodities, futures, contingent claims, endowments, technologies, and information) will guarantee efficiency.

Which combinations of markets will suffice in various circumstances is a critical question for policy.

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NOTES

1. The original article is M. C. Weinstein and R. J. Zeckhauser, "The Optimal Consumption of Depletable Natural Resources," this *Journal,* LXXXIX (Aug. 1975), 371–392.

2. If two or more firms are mining the same deposit simultaneously, it can be assumed that each firm ignores the effect of current extraction by other firms on x^k.