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ECONOMIC INCENTIVES FOR POLLUTION CONTROL

Maureen L. Cropper

1. INTRODUCTION. All economies must by some means determine what resources to devote to pollution control. When the number of polluters and victims is small this decision is often reached through a bargaining process. Bargaining, however, becomes costly as the number of parties increases. Pollution problems involving many polluters or victims are therefore solved by a neutral party, the government, which imposes penalties on polluters to minimize the total costs of pollution.

In either case determining the appropriate level of pollution requires three types of information: the costs to firms of reducing emissions, the damages to victims associated with ambient pollution, and the relationship between emissions and ambient pollution. Given this information, and assuming that emissions are costlessly observable, it is a simple matter to design an optimal contract in the bargaining case, or an optimal regulatory scheme when the government controls pollution, which is a function of emissions.

What makes the pollution control problem difficult is that the necessary information is often known imperfectly. The purpose of these notes is to examine how incentives for pollution control should be structured depending on what is known about damages, abatement costs, and emissions.

Section 2 of the notes models agreements between a single polluter and a single victim, either one of whom may initially be assigned rights to the environment. When abatement costs, damages, and emissions are known to both parties one can design a contract which improves the welfare of at least one party (a Pareto optimal contract) regardless of the initial assignment of property rights. In many situations, however, emissions can be monitored by the victim only at great cost. If the relationship between emissions and damages is also known imperfectly, incentives cannot be based on emission levels. One question in this case is whether the first-best solution to the problem—the solution which would be reached if emissions were costlessly observable—can be reached with imperfect information. When a first-best solution cannot be reached the best solution achievable can be characterized.

Section 3 considers many-firm pollution problems, which are typically solved by the government imposing penalties on polluters. Here a first-best solution may also be unachievable due to the high cost of monitoring emissions, and the questions addressed in Section 2 arise again. A different information problem, however, becomes important in the many-firm case. Since the government must regulate thousands of firms in diverse industries it is expensive to obtain information about all firms' abatement costs. The government can, of course, request that firms provide this information; however, if firms suspect that the information will be used to set pollution taxes they will have an incentive to misrepresent costs. A natural question is whether a system of taxes can be devised which will induce firms to correctly reveal their cost functions. The answer, under certain conditions, is yes; however, the cost of communication required to elicit this information may be prohibitive. In this case one can view the government's ignorance about costs as a constraint and characterize the best solution achievable given this information.

2. OPTIMAL POLLUTION REGULATION VIA BARGAINING. The two-party pollution problem may be illustrated by two firms, a farm and a papermill, located on a river. The papermill, located upstream from the farm, wishes to use the river to dispose of organic waste, while the farm wishes to use the river for irrigation. Suppose that emissions e, $e \in R^1_+$, are a byproduct of an m-dimensional vector of outputs $\underline{y}$, $\underline{y} \in R^m_+$, produced by the papermill. Let $G(\underline{y},e)$ denote the minimum total cost of producing a given $(\underline{y},e)$ vector and let $H(\underline{y})$ denote the revenue which the firm receives from a given output vector. The cost to the firm of emitting e of pollution is defined to be the difference between maximum profits if emissions are unconstrained and maximum profits if emissions are e,

$$C(e) \equiv \max_{\underline{y},e}[H(\underline{y})-G(\underline{y},e)] - \max_{\underline{y}}[H(\underline{y})-G(\underline{y},e)]. \tag{1}$$

We assume that H is a concave function of $\underline{y}$ and that G is a strictly convex function of $(\underline{y},e)$ so that $C(e)$ is a strictly convex function of e. We also assume that $C'(e) \le 0$ for $0 \le e \le \bar{e}$, where $\bar{e}$ is the level of emissions which the papermill would choose if unconstrained, i.e., $C(\bar{e}) = 0$. Since $C(e)$ is the cost of reducing emissions from $\bar{e}$ to e, C will sometimes be referred to as the cost of pollution abatement rather than the cost of emissions.

Damages to the farm from water pollution depend not on emissions themselves but on ambient water quality. Let x, $x \in R^1_+$, represent ambient water pollution and define damages D associated with x as the difference between the farm's profits when pollution is x and profits in the absence of pollution,

$$D = D(x), \quad D'(x) > 0, \quad D''(x) > 0. \tag{2}$$

2.1. BARGAINING UNDER PERFECT INFORMATION. Suppose in this example that the cost and damage functions are known to both firms, and that ambient pollution is a strictly increasing, convex function of e, $x = x(e)$, which is also known by both firms. If e is costlessly observable by both parties, or if x is observable so that e can be inferred from $x(e)$, then it is easy to construct an optimal pollution contract, i.e., one that makes at least one party better off compared with the pre-contract situation.

Let $C_0 \equiv C(e_0)$ denote abatement costs paid by the polluter in the pre-contract situation and $D_0 \equiv D(x(e_0))$ denote damages initially suffered by the victim. If at some $0 \le e \le \bar{e}$, $e \ne e_0$ the net change in abatement costs plus damages is positive,

$$C_0 - C(e) + D_0 - D(x(e)) > 0, \quad 0 \le e \le \bar{e}, \tag{3}$$

opportunities exist for a mutually beneficial agreement. Let $S(e)$ denote the amount which the victim pays the polluter to set emissions at e. (If $S(e) < 0$ then the polluter pays the victim.) A Pareto optimal contract is a function S which maximizes the bargaining gains to one party, e.g., the victim, subject to the constraint that the other party be no worse off than in the pre-contract situation. Formally, $S(e)$ must satisfy

$$\begin{aligned} &\max_S \; D_0 - D(x(e)) - S(e) \\ &\text{s.t. } \; S(e) - C(e) \ge -C_0 \;\text{ and }\; e = \operatorname{argmax}[S(e)-C(e)]. \end{aligned} \tag{4}$$

It is easily verified that (5) constitutes a Pareto optimal contract,

$$\begin{aligned} S(e) &= D_0 - D(x(e)) - k, \\ k &= D_0 - D(e^*) + C_0 - C(e^*), \end{aligned} \tag{5}$$

where e^* is the value of e that maximizes the left-hand side of (3). Equation (5) induces the polluter to set emissions at the level that minimizes damages plus abatement costs and divides net benefits according to initial bargaining positions.

For (5) to be enforceable, however, both parties must know the functions $D(x)$, $C(e)$, and $x(e)$ and must be able to observe x or e. These conditions will not be met if emissions are impossible to monitor, as in the case of agricultural runoff. Inability to monitor e presents no problem if x is cheaply observable and $x(e)$ known; however, the relationship between emissions and ambient pollution may be uncertain as well. This is precisely the situation which would obtain if the polluter were a farm and the victim a fisherman. Section 2.2 defines and characterizes a Pareto optimal contract under these circumstances.

2.2. BARGAINING UNDER IMPERFECT INFORMATION. To focus on the problem of monitoring emissions suppose that both firms know $C(e)$ and $D(x)$ but regard the relationship between ambient pollution and emissions as uncertain. Formally, ambient pollution is a function of emissions and a random variable θ, $\theta \in R^1$, where θ might represent water temperature or stream flow,

$$x = x(e,\theta), \quad \partial x/\partial e > 0, \quad \partial^2 x/\partial e^2 > 0, \quad \text{all} \quad \theta. \tag{6}$$

At the time the contract is negotiated both firms have identical probability distributions on θ. After the contract is negotiated the realized value of θ may be observed by one or both parties. In Model 1 the polluter remains uncertain about θ when e is chosen; in Model 2 the realized value of θ is known by him before he chooses e. In either case the realized value of θ may be observable by both parties after x occurs so that a contract can be based on θ as well as x.

We now consider the form which pollution contracts may take. Let $\underline{z}$ denote the variables observable by both parties after e has been chosen and θ realized. In this section $\underline{z}$ cannot include e but includes x and may include the realized value of θ. Let $S(\underline{z})$ be the payment made by the victim to the polluter. A Pareto optimal contract is defined as a function $S(\underline{z})$ which maximizes expected utility of benefits to the victim subject to the constraint that the expected utility of benefits to the polluter not fall below their pre-contract level, and to the constraint that the value of emissions chosen maximizes the expected utility of the polluter. Formally, the function $S(\underline{z})$ must satisfy (7)-(9),

$$\max_{S} E_\theta U^V[D_0 - D(x(e,\theta)) - S(\underline{z})] \equiv V^V(S(\underline{z}),e) \tag{7}$$

$$\text{s.t.} \qquad E_\theta U^P[S(\underline{z}) - C(e)] \geq U^P(-C_0) \tag{8}$$

$$\text{and} \qquad \max_{e} E_\theta U^P[S(\underline{z}) - C(e)] \equiv V^P(S(\underline{z}),e) \qquad \text{(Model 1)} \tag{9a}$$

$$\text{or} \qquad \max_{e} E_\theta U^P[S(\underline{z}) - C(e(\theta))] \equiv V^P(S(\underline{z}),e) \qquad \text{(Model 2).} \tag{9b}$$

U^V and U^P, the utility functions of the victim and the polluter, are assumed to be strictly increasing and concave. Equations (9a) and (9b) are assumed to have unique solutions.

This definition of Pareto optimality differs in two respects from (4). Equation (4) treats the functions U^V and U^P as linear, implying that

$$E_\theta U[D(\theta)] = U[E_\theta(D(\theta))], \quad (10)$$

i.e., that both firms are risk-neutral. If (10) holds then neither firm requires compensation for uncertainty about pollution, a reasonable assumption if each firm is owned by a large number of investors with diversified portfolios. If, however, a firm is closely-held and if uncertainty about pollution damages is large it may be reasonable to assume that the firm is risk-averse,

$$E_\theta U[D(\theta)] < U[E_\theta(D(\theta))], \quad (11)$$

and requires compensation for uncertainty regarding the outcome of the bargaining agreement. In this case a Pareto optimal contract must share this risk between the polluter and victim, as well as providing incentives for the polluter to alter his emissions.

Secondly, since the pollution contract cannot be based on emissions, equation (9a) or (9b) may be binding. A first-best contract is defined as one in which e and S are chosen to maximize (7) subject only to (8), i.e., a contract which could be realized if e were observable.

Given these definitions there are two questions of interest: (1) When emissions are not observable can a first-best contract be achieved? (2) If such a solution cannot be achieved, what does a Pareto optimal contract look like? Answers to both questions are provided by the literature on the principal-agent problem (Harris and Raviv [4], Holmstrom [5], Shavell [8]), of which the pollution problem is a specific example.

As shown by Harris and Raviv [4], there are two conditions under which a first-best solution is achievable when emissions cannot be observed. One is when the realized value of θ is observable by both parties _ex post_ so that the contract can be based on θ as well as on x. The other occurs when θ is not observable by both parties but the polluter is risk-neutral. These results, which hold for models 1 and 2, are stated formally as

PROPOSITION 1. Any contract based on $\underline{z} = (x\,\theta\,e)$ can be dominated by a contract based on $\underline{z} = (x\ \theta)$ in the sense that $V^V(S(\underline{z}),e)$ and $V^P(S(\underline{z}),e)$ are at least as great for both parties when $\underline{z} = (x\ \theta)$.

PROPOSITION 2. If the polluter is risk-neutral any contract based on $\underline{z} = (x\,\theta\,e)$ can be dominated by a contract based on $z = x$.

An important implication of these propositions is that monitoring emissions is of no value when θ is observable or the polluter is risk-neutral.

In the farm-fisherman example a first-best contract is achievable as long as ambient water quality can be cheaply monitored and weather conditions influencing water quality, assuming they are the source of θ, are also observable. The other case in which monitoring of emissions is unnecessary is when the polluter maximizes expected profits. In this case there is a Pareto optimal contract (Shavell [8, Prop. 4]) which pays the polluter $D_0 - D - k$ and the victim k. This solves the incentive problem by forcing the polluter to consider the effect of his emissions on the victim and provides for optimal risk sharing, i.e., the risk-neutral polluter bears all the risk.

When the polluter is risk-averse and θ is not observable ex post the Pareto optimal contract is no longer a first-best contract and there are gains to monitoring emissions. Since perfect monitoring is by assumption too costly, it is natural to ask whether an optimal contract can be improved upon by imperfect monitoring. The victim, for example, may be unable to observe emissions but may have information about a related variable, y, e.g. stream color. Suppose that y is a function of e and a random variable δ. Should the pollution contract be based on y given that this increases uncertainty about the payoff and given that the polluter is risk-averse? For Model 1 Shavell [8, Prop. 5] has shown that as long as the distribution of y depends on e, a contract based on x and y exists which dominates a contract based solely on x. In the special case in which $y = e + \delta$, where δ is distributed independently of θ on the interval $[\delta_0, \delta_1]$, $\delta_0 < 0 < \delta_1$, Harris and Raviv [4] demonstrate that the optimal contract is dichotomous,

$$S(x,y) = \begin{cases} S_0(x,y) & \text{if} \quad y \le \hat{y} \\ w & \text{if} \quad y > \hat{y} \end{cases}, \tag{12}$$

i.e., a payment which is a continuous function of ambient pollution and output is received if output falls below a critical level and a constant amount is received if output exceeds that level. This implies, in particular, that if e is costlessly observable a first-best solution can always be attained by means of a dichotomous (forcing) contract.

3. POLLUTION CONTROL WHEN BARGAINING IS NOT POSSIBLE. When the number of polluters and/or victims is large, voluntary agreements of the type described in Section 2 are too costly to arrange and pollution is usually regulated by a neutral third party, the government. Even under full information this changes the structure of the pollution problem considerably. Pollution regulation is no longer a voluntary agreement between two parties but a set of penalties imposed on firms which may reduce their utility below that received in the

absence of regulation. The definition of an optimal set of penalties is, however, similar to the definition of a Pareto optimal contract in the bargaining case. In Section 2 a necessary condition for a contract to be Pareto optimal is that it minimizes $D(x) + C(e)$. In the multi-firm case an optimal set of penalties is that which minimizes the sum of control costs plus damages.

As in the two-person problem, it is convenient to describe the solution to the multi-firm pollution problem under full information. Section 3.2 considers how this solution is altered when emissions cannot be observed, while Section 3.3 focuses on the government's ignorance of firms' cost functions.

3.1. POLLUTION CONTROL UNDER FULL INFORMATION. Consider n firms located in the same geographic area, each of which emits an amount e_i, $i = 1,\ldots,n$, of a pollutant such as particulate matter or sulfur dioxide. The ambient level of the pollutant, x, is a known, convex function of the vector $\underline{e} = (e_1 e_2 \ldots e_n)$, $x(\underline{e})$. Let x_i denote the partial derivative of this function with respect to e_i and assume $x_i > 0$, $i = 1,\ldots,n$. As in the bargaining case damages to society are assumed to be an increasing, strictly convex function of ambient pollution, $D = D(x)$. The cost of emissions e_i to firm i is given by $C(e_i,\alpha_i)$, where C is defined by equation (1). α_i, which could be a vector but for notational simplicity is treated as a scalar, is a parameter which distinguishes firm i's abatement cost function from firm j's. As in Section 2 C is assumed to be a strictly convex function of e_i for each value of α_i.

Given the functions C, D and x and the parameter vector $\underline{\alpha} = (\alpha_1 \ldots \alpha_n)$ it is a simple matter to determine the vector of emissions $\underline{e}^*$ which minimizes the sum of damages plus abatement costs,

$$\underline{e}^* = \operatorname{argmin}[D(x(\underline{e})) + \sum_{i=1}^{n} C(e_i,\alpha_i)]. \tag{13}$$

Previous assumptions guarantee that the solution to (13) is unique and is characterized by the necessary conditions

$$D'(x(\underline{e}^*))x_i(\underline{e}^*) = -C_1(e_i^*,\alpha_i) \quad i = 1,\ldots,n, \tag{14}$$

where $C_1(e_i,\alpha_i) \equiv \partial C/\partial e_i$.

Assuming that each firm chooses e_i to minimize the sum of abatement costs plus taxes, and that the government can costlessly observe emissions, the full-information solution to the pollution control problem can be achieved by levying on firm i a tax t_i per unit of emissions,

$$t_i = D'(x(\underline{e}^*))x_i(\underline{e}^*). \tag{15}$$

3.2. POLLUTION CONTROL WITH COSTLY MONITORING. As in the two-party case, the cost of monitoring emissions may be so great in the multi-firm case that one wishes to consider only the class of penalties based on ambient pollution levels. Since this topic has been treated in detail in Section 2 we emphasize ways in which the results of that section are altered by considering multiple firms.

In Section 2 the victim's inability to observe e prevents a first-best solution from being attained only if the relationship between ambient pollution and emissions is uncertain. With many firms removing this uncertainty does not solve the monitoring problem since any one firm's emissions cannot be inferred by observing x even if the function $x(\underline{e})$ is known. We therefore ask whether the full-information solution can be attained through penalties which depend only on x when $x(\underline{e})$ is known.

The answer, as shown by Holmstrom [6], is yes. Define T_i, the penalty on the ith firm, as

$$\begin{aligned} T_i &= 0 \quad \text{if} \quad x \le x(\underline{e}^*) \\ T_i &= b_i \quad \text{if} \quad x > x(\underline{e}^*) \end{aligned} \tag{16}$$

where $b_i > C(e_i^*,\alpha_i)$. It is easily verified that firm i minimizes $T_i + C(e_i,\alpha_i)$ by setting $e_i = e_i^*$, assuming that $e_j = e_j^*$ for all $j \neq i$. In other words $\underline{e}^*$ is a Nash (non-cooperative) equilibrium if penalties are set according to (16).

A similar solution is obtainable when all polluters are risk-neutral and the relationship between ambient pollution and emissions is uncertain. Let $F(\bar{x},\underline{e})$ be the probability that $x \ge \bar{x}$ given that emissions are $\underline{e}$, and assume that $\partial F/\partial e_i \equiv F_i(\bar{x},\underline{e}) > 0$, all i, and that $F(\bar{x},\underline{e})$ is a strictly convex function of $\underline{e}$. If penalties of the form

$$\begin{aligned} T_i &= 0 \quad \text{if} \quad x \le \bar{x} \\ T_i &= d_i \quad \text{if} \quad x > \bar{x} \end{aligned} \tag{17}$$

are imposed on firm i the firm will set

$$F_i(\bar{x},\underline{e})d_i + C_1(e_i,\alpha_i) = 0 \tag{18}$$

to minimize the expected value of abatement costs plus penalties. By choosing

$$d_i = -C_1(\hat{e}_i,\alpha_i)/F_i(\bar{x},\hat{\underline{e}}) \tag{19}$$

one guarantees that firm i sets $e_i = \hat{e}_i$ provided that all firms $j \neq i$ set $e_j = \hat{e}_j$; i.e., one guarantees that $\hat{\underline{e}}$ is a Nash equilibrium. A penalty of the form (17) can thus be used to achieve the emissions vector that minimizes expected damages plus abatement costs when $\underline{e}$ is unobservable and the relationship between ambient pollution and emissions is uncertain.

3.3. POLLUTION CONTROL UNDER IMPERFECT INFORMATION ABOUT FIRM TECHNOLOGY. To focus on the problem of monitoring emissions it has been assumed that control cost and damage functions are known by all parties. Although it is reasonable to assume that each firm knows its cost of emissions function, it is unlikely that the government knows $\underline{\alpha}$ with certainty. The government is also unlikely to know the damage function $D(x)$; however, the two information problems are fundamentally different. Since ignorance of $\underline{\alpha}$ is a problem of asymmetrical information one can ask whether the government can induce firms to truthfully report $\underline{\alpha}$ and thus achieve the full-information solution $\underline{e}^*$. Ignorance about $D(x)$, however, more likely represents scientific ignorance, e.g., ignorance about the health effects of pollution, rather than an information asymmetry. It is therefore natural to redefine the optimal pollution vector taking uncertainty about $D(x)$ as given. Since this will not alter the structure of optimal pollution penalties we concentrate instead on the problem of asymmetric information regarding firms' costs. To focus on this problem it is assumed that emissions are costlessly observable by all parties and that the $x(\underline{e})$ and $D(x)$ functions are known.

We begin by noting that the government cannot simply request that each firm report its α_i, for if the firm suspects that this information will be used to compute emissions taxes it will misreveal α. Specifically, if firms believe that taxes will be set according to (15) and if all firms affect ambient pollution symmetrically,

$$x = x\left(\sum_{i=1}^{n} e_i\right), \tag{20}$$

it can be shown that each firm has an incentive to understate its marginal cost of pollution abatement (Kwerel [7]).

To show this, suppose that α_i increases the marginal cost of pollution abatement,

$$-\partial^2 C/\partial e_i \partial\alpha_i > 0, \quad \text{all} \quad \alpha_i. \tag{21}$$

Let $\hat{\alpha}_i$ denote the value of α_i revealed to the government (α_i denotes the true parameter value), and $\underline{\hat{\alpha}}_{-i}$ denote the vector of parameters revealed by firms other than i. $\hat{e}_i$ denotes the value of emissions which minimizes firm i's taxes plus abatement costs, $t_i e_i + C(e_i, \alpha_i)$, if $\underline{\hat{\alpha}}$ is reported to the government. Since

$$\partial[t_i\hat{e}_i+C(\hat{e}_i,\alpha_i)]/\partial\hat{\alpha}_i = (\partial t_i/\partial\hat{\alpha}_i)\hat{e}_i \tag{22}$$

the firm clearly has an incentive to set $\hat{\alpha}_i < \alpha_i$ if t_i is an increasing function of $\hat{\alpha}_i$ for any $\underline{\hat{\alpha}}_{-i}$.

To see that $\hat{\alpha}_i$ increases t_i note that when (20) holds (14) reduces to

$$D'(\sum \hat{e}_i^*) = -C_1(\hat{e}_i^*,\hat{\alpha}_i) \quad i = 1,\ldots,n, \tag{23}$$

where $\underline{\hat{e}}^*$ denotes the optimal emissions vector given that firms reveal $\underline{\hat{\alpha}}$. In this case all firms optimally produce at the same marginal emissions cost and tax rates facing all firms are identical,

$$t_i = t = D'(\sum \hat{e}_i^*), \text{ all } i. \tag{24}$$

That $\hat{\alpha}_i$ raises t follows from (23) and (24) which together imply

$$\frac{\partial t}{\partial\hat{\alpha}_i} = D'' \sum_{j=1}^{n} \frac{\partial\hat{e}_j}{\partial\hat{\alpha}_i} = -D'' \frac{\partial^2 C/\partial e_i\partial\hat{\alpha}_i}{\partial^2 C/\partial e_i^2+D''} > 0. \tag{25}$$

In view of this result the question is whether there exist taxes different in form from (15) which will induce firms to reveal their true α_i's. An affirmative answer to this question has been provided by Groves and Loeb [3]. To define the tax levied on firm i recall that $\hat{e}_j^*$ denotes optimal emissions for firm j given that firms report $\underline{\hat{\alpha}}$ to the government. To emphasize that $\hat{e}_j^*$ is a function of firm i's reported α, $\hat{\alpha}_i$, as well as of the values of α reported by all other firms, write

$$\hat{e}_j^* = \hat{e}_j^*(\hat{\alpha}_i,\underline{\hat{\alpha}}_{-i}). \tag{26}$$

The Groves-Loeb tax paid by firm i, which depends both on $\hat{\alpha}_i$ and on e_i, is defined as

$$\begin{aligned} T_i(e_i,\hat{\alpha}_i,\underline{\hat{\alpha}}_{-i}) &= D(x(e_i,\underline{\hat{e}}^*_{-i}(\hat{\alpha}_i,\underline{\hat{\alpha}}_{-i}))) \\ &\quad + \sum_{j\neq i} C(\hat{e}_j^*(\hat{\alpha}_i,\underline{\hat{\alpha}}_{-i}),\hat{\alpha}_j) + A_i(\underline{\hat{\alpha}}_{-i}) \end{aligned} \tag{27}$$

where $\underline{\hat{e}}^*_{-i}$ denotes the optimal vector of emissions for all firms other than i and A_i is a firm-specific function.

For the tax functions (27) to induce truthful reporting of α_i regardless of the parameters reported by other firms it must be the case that

$$\begin{aligned} &C(\hat{e}_i(\alpha_i, \underline{\hat{\alpha}}_{-i}),\alpha_i) + T_i(\hat{e}_i(\alpha_i,\underline{\hat{\alpha}}_{-i}),\alpha_i,\underline{\hat{\alpha}}_{-i}) \\ &\leq C(\hat{e}_i,\alpha_i) + T_i(\hat{e}_i,\hat{\alpha}_i,\underline{\hat{\alpha}}_{-i}) \quad \text{all } \hat{\alpha}_i \neq \alpha_i. \end{aligned} \tag{28}$$

To see that this is so, note that from the definition of $\hat{e}_j^*$ and T_i

$$\begin{aligned}
& C(\hat{e}_i(\alpha_i,\hat{\underline{\alpha}}_{-i}),\alpha_i) + T_i(\hat{e}_i(\alpha_i,\hat{\underline{\alpha}}_{-i}),\alpha_i,\hat{\underline{\alpha}}_{-i}) - A_i(\hat{\underline{\alpha}}_{-i}) \\
&= \min_{\underline{e}_{-i}}[D(e_i(\alpha_i,\hat{\underline{\alpha}}_{-i}),\underline{e}_{-i}(\alpha_i,\hat{\underline{\alpha}}_{-i})) \\
&\qquad + \sum_{j\neq i} C(e_j,\hat{\alpha}_j) + C(e_i,\alpha_i)] \\
&\leq D(e_i(\hat{\alpha}_i,\hat{\underline{\alpha}}_{-i}),\underline{e}_{-i}(\hat{\alpha}_i,\hat{\underline{\alpha}}_{-i})) \\
&\qquad + \sum_{j\neq i} C(e_j,\hat{\alpha}_j) + C(e_i,\alpha_i) \\
&= C(\hat{e}_i,\alpha_i) + T_i(\hat{e}_i,\hat{\alpha}_i,\hat{\underline{\alpha}}_{-i}) - A_i(\hat{\underline{\alpha}}_{-i}).
\end{aligned} \tag{29}$$

Thus for any $\hat{\underline{\alpha}}_{-i}$, firm i's dominant strategy is to report α_i.

There are, however, two drawbacks to the Groves-Loeb tax. Under the tax it is optimal for firm i to report its true α_i only if the firm cannot communicate with other firms. There is in general no tax scheme which will induce true revelation of information if firms can form coalitions (Green and Laffont [2]).

Secondly, for the government to impose a tax which achieves the first-best solution two rounds of communication are required. In round one the government sends firms tax functions (27) and receives the parameter vector $\underline{\alpha}$. During round two the $\underline{\alpha}$ vector is communicated to each firm, and taxes are recomputed with $\hat{\underline{\alpha}}_{-i}$ replaced by $\underline{\alpha}_{-i}$. If such communication is not possible then the first-best solution cannot be achieved (Dasgupta, Hammond and Maskin [1]).

This implies that one must adopt an alternative definition of an optimal incentive structure if the government must announce taxes based only on its initial information. To formalize what is known about $\underline{\alpha}$ suppose that all firms and the government view the $\{\alpha_i\}$ as independent drawings from a publicly known probability distribution. The realized value of α_i is known only to firm i. In defining the optimal set of taxes suppose that firm i can be taxed only on its own emissions, e_i, since to tax the firm based on $\underline{e}_{-i}$ would subject it to uncertainty regarding other firms' costs. Let $\tilde{e}_i$ denote firm i's optimal response to the tax function $T_i(e_i)$, i.e.,

$$\tilde{e}_i = \operatorname{argmin} T_i(e_i) + C_i(e_i,\alpha_i). \tag{30}$$

The optimal tax functions $T_i(e_i)$ are those which minimize the sum of expected damages plus pollution costs,

$$E[D(\tilde{e}_1(T_1(\ ),\alpha_1),\ldots,\tilde{e}_n(T_n(\ ),\alpha_n)) + \sum_{i=1}^{n} C_i(\tilde{e}_i(T_i(\ ),\alpha_i),\alpha_i)], \tag{31}$$

given firms' response functions.

In the case in which the marginal damage function is linear in $\underline{e}$ and each marginal emission cost function is linear in e_i and α_i Weitzman [9] has shown that the optimal tax is a combination of a per-unit tax on emissions, t_i, and a penalty for deviating from the emissions quota e_i^*,

$$T(e_i) \quad = \quad t_i e_i + q_i(e_i - e_i^*)^2. \tag{32}$$

The emissions quotas $\{e_i^*\}$ are those levels of e_i that minimize the sum of expected damages plus control costs,

$$\underline{e}^* \quad = \quad \text{argmin } E[D(\underline{e}) + \sum_{i=1}^{n} C_i(e_i,\alpha_i)]. \tag{33}$$

t_i is the marginal damage caused by firm i when $\underline{e} = \underline{e}^*$,

$$t_i \quad = \quad \partial D(\underline{e})/\partial e_i \Big|_{\underline{e}=\underline{e}^*}, \quad i = 1,\ldots,n. \tag{34}$$

The cost of the firm of deviating from the expected-loss-minimizing emissions level depends on the slope of the marginal damage function. Since $\partial D/\partial e_i$ is assumed linear in $\underline{e}$ we may write

$$\partial D/\partial e_i \quad = \quad t_i - \sum_{j=1}^{n} \beta_{ji}(e_j - e_j^*), \quad i = 1,\ldots,n, \tag{35}$$

where t_i is defined by (34). q_i varies directly with β_{ii}, the slope of the marginal damage function with respect to firm i's emissions,

$$q_i \quad = \quad \beta_{ii}/2. \tag{36}$$

Under the foregoing assumptions the optimal taxation scheme has an intuitively appealing interpretation. When marginal damages increase slowly with emissions the optimal penalty is a linear tax on emissions. Although it is uncertain how much this tax will reduce emissions, the cost of deviating from the optimal emission level is small since damages increase slowly with emissions. When the marginal damage function is steep, however, deviations from $\underline{e}^*$ are costly to society and the firm is therefore penalized heavily for deviating from the optimal emissions level.

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DEPARTMENT OF ECONOMICS
UNIVERSITY OF MARYLAND
COLLEGE PARK, MARYLAND 20742