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Estimation of a variable rate of depreciation: a dummy variable approach

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Abstract

Many important economic problems require measures of both physical and R&D capital. Except for some recent studies, there have been relatively few contributions in the literature that provide econometric estimates for the depreciation rates of physical and R&D capital. One reason for the relative paucity of such econometric studies may be that if the depreciation rates are viewed as unknown parameters, or as a functions of unknown parameters, then the corresponding stocks of physical and R&D capital are also unobserved, resulting in a formidable estimation problem. In the past, econometric estimates of the depreciation rates of physical and R&D capital were typically obtained from self-programmed estimation algorithms. In this note, an approach is introduced that permits the econometric estimation of constant and variable rates of depreciation of physical and R&D capital using standard econometric packages. © 1997 Elsevier Science B.V.

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1. Introduction[1]

Many important economic problems require measures of capital. For example, physical capital plays a pivotal role in studies on production and cost, and in studies on the sources of productivity and output growth. In addition, knowledge capital approximated by R&D capital has been recognized as an important contributor to productivity growth. Furthermore, for policy purposes it is important to be able to distinguish between net and replacement investment in these types of capital. Crucial

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to an analysis of the contributions of physical and R&D capital is the measurement of the stocks of physical and R&D capital, which, in turn, typically requires measuring their depreciation rates. The conventional procedure for computing estimates of the stocks of physical and R&D capital is the perpetual inventory method; see, for example, Hulten (1990) for a recent review. While the perpetual inventory method permits the incorporation of detailed information/assumptions (concerning the mean useful life of assets, the retirement distribution centered on that life, and on efficiency patterns), these assumptions, and hence the implied estimates of the depreciation rates, are typically not subjected to rigorous econometric testing. Recently, however, there have been several studies that provide econometric estimates of the depreciation rates of the stocks of physical and/or R&D capital goods; see, for example, Epstein and Denny (1980), Kokkelenberg (1984), Kollintzas and Choi (1985), Bischoff and Kokkelenberg (1987), Nadiri and Prucha (1996) and Prucha and Nadiri (1996).

One reason for the relative paucity of such econometric studies may be that if the depreciation rates are viewed as unknown parameters, or as functions of unknown parameters, then the corresponding stocks of physical and R&D capital are also unobserved, resulting in a formidable estimation problem. Of course we can, in principle, always express the capital stocks as functions of the observed current and past investments and the unknown depreciation rates, substitute those functions for the stocks in, say, the production function or a set of factor demand equations, and then estimate the resulting relationships. However, in addition to the resulting complexity of the estimation problem, we then also encounter an implementation problem if we try to use standard econometric packages for this task. Estimation routines as programmed in standard econometric packages (such as, for example, the LSQ procedure in TSP) typically do not permit the specification of functions where the argument list, say, increases with time. However, as discussed in more detail below, this is exactly the case if stocks are generated recursively from some initial stock in the usual way. Therefore, all of the above-cited studies that report econometric estimates of depreciation rates have not used standard econometric packages, but found it necessary to program their own estimation routines, which is time consuming and requires specialized skills (or to settle for an approach that does not generate a fully consistent capital stock series). In the following we now show how a reformulation of the estimation problem, using dummy variables, permits the application of standard econometric packages. The approach discussed below will cover a variety of applications including not only constant but also variable depreciation rates, including situations where the depreciation rate of one capital good depends on the stock of some other captial good. While, on the one hand, the observations made below are simple, it still seems that, on the other hand, they were not evident in the past.

2. An illustration of the problem

In the following we give a simple example that illustrates the practical difficulties in econometrically estimating capital depreciation rates. Suppose a researcher would like to estimate the following production function:

$$Y_t = F(L_t, K_t, R_t, \theta), \qquad t=1, \dots, T, \tag{1}$$

where Y_t and L_t denote, respectively, output and labor input in period t, where K_t and R_t denote stocks of two types of capital, e.g. physical and R&D capital, at the end of period t, and where T denotes the sample size. With θ we denote the vector of unknown model parameters. For simplicity, assume for the moment that only the depreciation rate of K_t is unknown and hence K_t is unobserved, but that R_t is observed. Assume further, for the moment, that the depreciation rate of K_t, say δ^K, is constant. The capital stock K_t then accumulates according to the equation:

$$K_t = I_t^K + (1-\delta^K)K_{t-1}, \tag{2}$$

where I_t^K denotes gross investment. Solving Eq. (2) recursively yields:

$$K_t = k_t(I_t^K, \dots, I_1^K, K_0, t, \delta^K) = \sum_{i=0}^{t-1} I_{t-i}^K(1-\delta^K)^i + K_0(1-\delta^K)^t. \tag{3}$$

Substitution of this expression into the production function Eq. (1) gives:

$$Y_t = F(L_t, k_t(I_t^K, \dots, I_1^K, K_0, t, \delta^K), R_t, \theta) = G_t(L_t, I_t^K, \dots, I_1^K, K_0, R_t, t, \delta^K, \theta). \tag{4}$$

Note that $L_1,\dots,L_T$ and $I_1,\dots,I_T$ are observed and thus, in principle, we can estimate the production function parameters θ and the depreciation rate δ^K jointly from Eq. (4) by some standard estimation methods for nonlinear models.[2] However, as remarked above, there is a practical difficulty if we would like to apply a standard econometric package such as, for example, TSP. The argument list of $k_t(\cdot)$, and hence of the function $G_t(\cdot)$, depends on t. However, as remarked above, standard econometric packages will typically not permit the specification of a function where the argument list changes with t; they can hence not be employed directly to estimate Eq. (4).

3. A dummy variable approach

In the following we now show how, for a fairly wide class of models of capital accumulation, we can 'artificially' re-write the recursive solution for the capital stock such that the argument list does not depend on t. We first consider the case where

[2] For a survey of estimation methods for dynamic nonlinear models see, e.g. Pötscher and Prucha (1991a,b). We note that if an estimate of the initial stock K_0 is not available, we may also treat the initial stock as an unknown parameter.

only K_t is unobserved and accumulates according to the equation:

$$K_t = I_t^K + K_t^o, \qquad K_t^o = f(Z_t, \theta, \alpha) + a(Z_t, \theta, \alpha)K_{t-1}, \tag{5}$$

where K_t^o is the capital stock left over at the end of period t from the initial stock K_{t-1}, and where $f(\cdot)$ and $a(\cdot)$ are real valued functions that depend (possibly) on some vector of observed variables Z_t and (possibly) on the parameter vectors θ and α.[3] The parameter vector α was included to allow for parameters that are not included in θ. Solving Eq. (5) recursively for K_t yields:[4]

$$K_t = k_t(I_t^K, \ldots, I_1^K, K_0, Z_t, \ldots, Z_1, \theta, \alpha) = \sum_{i=0}^{t-1} (I_{t-i}^K + f_{t-i}) \prod_{j=0}^{i-1} a_{t-j} + K_0 \prod_{j=0}^{t-1} a_{t-j}, \tag{6}$$

where we use the abbreviations $f_t = f(Z_t, \theta, \alpha)$ and $a_t = a(Z_t, \theta, \alpha)$.

The motivation for the specification in Eq. (5) is that it contains various models that have appeared in the literature as special cases:[5] by setting $f(Z_t, \theta, \alpha) \equiv 0$ and $a(Z_t, \theta, \alpha) \equiv \alpha$, Eq. (5) covers the case of a constant depreciation rate with $\delta^K = 1 - \alpha$, which has, for example, been considered in Nadiri and Prucha (1996). By setting $f(Z_t, \theta, \alpha) \equiv 0$ and $a(Z_t, \theta, \alpha) = 1 - Z_t'\alpha$ we obtain a model where the depreciation rate depends linearly on Z_t. Suppose $Z_t = (1, t)'$, then the depreciation rate is a linear function of t, or if $Z_t = (1, CU_t)$, where CU_t denotes a measure of capacity utilization, then we obtain the specification considered by Bischoff and Kokkelenberg (1987). The models considered in Epstein and Denny (1980), Kollintzas and Choi (1985) and Prucha and Nadiri (1996) are also readily seen to be special cases of Eq. (5) with $a(Z_t, \theta, \alpha) = \alpha$ and where $f(Z_t, \theta, \alpha)$ represents some function of prices and outputs.

We now define the following dummy variable:

$$D_t = \begin{cases} 1 & 1 \le t \le T, \\ 0 & -T \le t < 1. \end{cases}$$

We furthermore assign arbitrary values to Z_t for $t < 1$ such that f_t and a_t remain well defined. Thus, we can rewrite Eq. (6) as:[6]

$$\begin{aligned} K_t &= k(I_t^K, \ldots, I_{t-T+1}^K, K_0, Z_t, \ldots, Z_{t-T+1}, D_t, \ldots, D_{t-T+1}, \theta, \alpha) \\ &= \sum_{i=0}^{T-1} (I_{t-i}^K + f_{t-i}) D_{t-i} \prod_{j=0}^{i-1} a_{t-j} + K_0 \prod_{j=0}^{T-1} a_{t-j}^{D_{t-j}}. \end{aligned} \tag{7}$$

[3] The depreciation rate of capital is then given by $\delta_t^K = 1 - K_t^o/K_{t-1}$, and will, in general, be variable over time.

[4] In the following expression we adopted the convention that $\Pi_{j=0}^{-1} a_{t-j} = 1$.

[5] The dimensionality of α varies in the subsequent eamples, but will be obvious from the context.

[6] In the following expression we have used implicitly $x^0 = 1$ for any real number x.

We note that the number of arguments of $k(\cdot)$ no longer depends on t.[7] Thus, the expression in Eq. (7) can be programmed in standard econometric packages and then substituted into some other equation, such as, for example, Eq. (1), for purposes of estimation. If $a_t \equiv a$, then the product expressions in Eq. (7) can be simplified to $\Pi_{j=0}^{i-1} a_{t-j} = a^i$ and $\Pi_{j=0}^{T-1} a_{t-j}^{D_{t-j}} = a^t$. Of course, if R_t is also unobserved, and it accumulates analogously to Eq. (5), and there is no interaction between K_t and R_t in the accumulation equations, then the same approach can be applied for R_t.

We next consider the case where both K_t and R_t are unobserved, and where the accumulation equation for K_t depends both on K_{t-1} and R_{t-1}. In particular, we consider the case where K_t and R_t accumulate according to the following equations:

$$K_t = I_t^K + K_t^o, \qquad K_t^o = f(Z_t, \theta, \alpha) + aK_{t-1} + cR_{t-1}, \tag{8a}$$

$$R_t = I_t^R + R_t^o, \qquad R_t^o = g(Z_t, \theta, \alpha) + bR_{t-1}. \tag{8b}$$

The function $g(\cdot)$ is defined analogously to $f(\cdot)$, and a, b and c represent scalars that may be elements of α or θ. The above accumulation equations again cover various models of interest. They contain, for example, the set of accumulation equations considered in Prucha and Nadiri (1996) as a special case.[8] It can be checked that the recursive solution for K_t and R_t is now given by:

$$K_t = \sum_{i=0}^{t-1} \left[(I_{t-i}^K + f_{t-i})a^i + (I_{t-i}^R + g_{t-i}) \frac{(a^i - b^i)c}{(a-b)} \right] + K_0 a^t + R_0 \frac{(a^t - b^t)c}{(a-b)}, \tag{9a}$$

$$R_t = \sum_{i=0}^{t-1} (I_{t-i}^R + g_{t-i})b^i + R_0 b^t, \tag{9b}$$

where we use the abbreviations $g_t = g(Z_t, \theta, \alpha)$. Of course, the above expression is only well defined for $a \neq b$; however, $(a^t - b^t)/(a-b) = \Sigma_{s=0}^{t-1} a^{t-1-s} b^s$. The latter expression is also appropriate for $a = b$, but more complex to implement. The solution in Eqs. (9a) and (9b) is best found by rewriting Eqs. (8a) and (8b) in matrix notation and by observing that the following diagonalization holds:

$$\mathbf{A} = \begin{bmatrix} a & c \\ 0 & b \end{bmatrix} = \mathbf{Q}\Lambda\mathbf{Q}^{-1}, \qquad \mathbf{Q} = \begin{bmatrix} a & -bc/(a-b) \\ 0 & b \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix},$$

[7] Of course, alternatively, we could write $K_t = \Sigma_{s=1}^{T} k_s D_t^s$ with $D_t^s = 1$ for $t = s$ and zero otherwise, and where k_s is defined in Eq. (6). However, the resulting expression for K_t would then typically be too complex to be practically implementable.

[8] We could have further generalized the specification by allowing a, b and c to vary over time and/or for K_{t-1} to also appear in the expression for R_t^o. However, it seems that then the algebraic expressions for the recusive solution will, in general, become too complex to be practically implementable. An alternative, but more complex approach for the case of a single capital good with a constant depreciation rate is discussed in Prucha (1995).

and thus $\mathbf{A}^i = \mathbf{Q}\Lambda^i\mathbf{Q}^{-1}$. Analogously to before, using the dummy variable D_t, we can rewrite Eqs. (9a) and (9b) as:

$$K_t = \sum_{i=0}^{T-1}\left[(I^K_{t-i}+f_{t-i})D_{t-i}a^i + (I^R_{t-i}+g_{t-i})D_{t-i}\frac{(a^i-b^i)c}{(a-b)}\right] + K_0a^t + R_0\frac{(a^t-b^t)c}{(a-b)}, \tag{10a}$$

$$R_t = \sum_{i=0}^{T-1}(I^R_{t-i}+g_{t-i})D_{t-i}b^i + R_0b^t. \tag{10b}$$

Again, in this formulation, the size of the argument lists on the right hand side of Eqs. (10a) and (10b) no longer depends on t, and hence the expressions can be programmed in standard econometric packages.

Of course, the above outlined dummy variable approach is not limited to two capital goods, but can also be applied to several capital goods, given their accumulation equations are of the form Eq. (5) and/or Eqs. (8a) and (8b). To test the feasibility of the dummy variable approach we applied the approach to re-estimate both the model in Nadiri and Prucha (1996) and that in Prucha and Nadiri (1996) using TSP. Originally the results reported in those papers were obtained via a self-programmed FORTRAN estimation program. In both cases we were able to duplicate the previously obtained results.

We also performed various speed comparisons between the self-programmed FORTRAN estimation program and the dummy variable approach using TSP.[9] We found that, on average, estimation runs based on the implementation of the dummy variable approach in TSP were four times faster than corresponding estimation runs that were based on the self-programmed FORTRAN estimation program. As an illustration, the results from the following experiment were typical: we re-estimated model 3 in Prucha and Nadiri (1996) by full information maximum likelihood using both approaches. (Model 3 allows for a variable depreciation rate and is the most general model considered in that paper.) In both cases we used the full information maximum likelihood parameter estimates reported in Prucha and Nadiri (1996) as starting values for the model parameters, except that we used a value of 1.0 rather than the reported estimate of 1.19 for the scale parameter ρ. The self-programmed FORTRAN estimation program took 135 seconds to converge back to the maximum of the likelihood function. In contrast, the dummy variable approach using TSP

[9] All speed comparisons were executed on an IBM PC 350 computer with a 133 Mhz Pentium processor, 32 Mb of memory, and OS/2 Warp as the operating system. The self-programmed FORTRAN estimation program utilizes the numerical optimization subroutine VA10AD from the Harwell program library. The version of the TSP program used was TSP 4.3 for OS/2. In both cases the user does not have to provide expressions for the derivatives of the objective function—which would be, in light of the complexity of the model, an enormously cumbersome task. As a technical point, we note that the VA10AD subroutine computes those derivatives by numerical differentiation. In contrast, the FIML routine in TSP—in addition to providing a much more user friendly environment—first calculates analytic derivatives and then evaluates those derivatives numerically.

converged in 30 seconds. Obviously, in applications we can typically not expect to start the estimation routine from such 'ideal' starting values as for this experiment. In fact, actual estimation runs that lead to the results reported in Prucha and Nadiri (1996) often took several hours until convergence was achieved. From this perspective the reported savings in computing time seem important.

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