Investor Sophistication and Capital Income Inequality*

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Abstract

We study the determinants of capital income inequality in a general equilibrium portfolio choice model with endogenous information acquisition. The key elements of the model are heterogeneity in investor sophistication and in asset riskiness. The model implies capital income inequality that increases with aggregate information technology, given initial heterogeneity in sophistication. The main mechanism in the model works through endogenous investor participation in assets with different risk. Across assets, the pattern of expansion of sophisticated investors and retrenchment of unsophisticated investors, unique to our model, is consistent with asset ownership dynamics for the U.S. Quantitatively, the model generates a path for capital income inequality that matches the evolution of inequality in U.S. data.

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The rise in wealth and income inequality worldwide has been one of the most hotly discussed topics in academic and policy circles.[1] A significant step towards understanding the patterns in the data is the vast literature on wage inequality and the polarization of the U.S. labor market.[2] Less understood thus far has been inequality in capital income generated in financial markets. An important component of total income, capital income is by far the most unequally distributed part of household income in the United States, and it exhibits a strong upward trend in polarization.[3]

A growing literature in economics and finance analyzes household behavior in financial markets and especially its impact on capital income.[4] Some of the robust general trends are a growing non-participation in risky investments and a decline in trading activity. Anecdotal evidence suggests that an ever present and growing disparity in investor sophistication, or access to investment technologies, might be partly responsible for these trends. An early articulation of this argument is Arrow (1987) and recently Piketty (2014). However, micro-founded, quantitative treatments of such mechanisms are missing.

We propose such a micro-founded, general equilibrium theory of portfolio choice that can go a long way in explaining the recent growth in capital income inequality, qualitatively and quantitatively. The friction in our model is heterogeneity in investor sophistication modeled as investors' ability to obtain and process information about their investments.

To explore the consequences of this friction for the dynamics of capital income inequality, we link initial sophistication to initial wealth. Intuitively, when information about financial assets is costly to process, individuals with different access to financial resources also differ in terms of their access to information about their financial investments. We take this point as a guiding principle in mapping investors in our model into two different wealth groups in the

[1]For a summary, see Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion is also in the 2013 Summer issue of the Journal Economic Perspectives and Piketty (2014).

[2]Representative contributions to this line of research include Katz and Autor (1999); Acemoglu (1999); Autor, Katz, and Kearney (2006, 2008); and Autor and Dorn (2013).

[3]In the U.S. Survey of Consumer Finances, approximately 34% of households participate in financial markets. Capital income accounts for approximately 14% of this group's total income, ranging from 35% to less than 1%. Between 1989 and 2013, the ratio of capital incomes for the top 10% of the financial wealth distribution relative to the bottom 50% increased from 61 to 129.

[4]Most recently represented by Calvet, Campbell, and Sodini (2007) and Chien, Cole, and Lustig (2011).

Survey of Consumer Finances. Specifically, in the population of households who participate in financial markets, we use the average financial wealth of the 10% wealthiest investors relative to that of the 50% poorest investors in 1989 as a proxy for *initial* relative investor sophistication. In the presence of this initial inequality, subsequent *symmetric growth* in the capacity to process information for both investor types disproportionately benefits the wealthy, more sophisticated investors. As a result, their wealth diverges from that of less wealthy investors, who have relatively less information. General equilibrium forces amplify this effect, as asset prices push the unsophisticated investors to allocate their investments away from the allocations of sophisticated investors, which results in further divergence. This process generates a path for capital income inequality that can quantitatively match the evolution of inequality in the data. A feedback mechanism, through which changes in financial wealth feed into subsequent investor sophistication, generates an endogenous evolution of capacity for each investor type that yields an even larger increase in inequality.

Formally, we build a noisy rational expectations equilibrium portfolio choice model with endogenous information acquisition and capacity constraints, in the spirit of Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). We generalize this theory by allowing for meaningful heterogeneity across both assets and investors. Our economy is populated with one riskless asset and many risky assets that differ in the volatilities of their fundamental payoffs. A continuum of investors have mean-variance preferences with a common risk aversion coefficient. Investors learn about assets payoffs from optimal private signals subject to an entropy constraint on information (Sims (2003)). Based on the observed asset characteristics, investors decide which assets to learn about, how much information about them to process, and how much wealth to invest. A fraction of investors are endowed with high capacity for processing information and the remaining ones have lower, yet positive capacity. Thus, everyone in the economy has the ability to learn about assets payoffs, but to different degrees.

Our methodological contribution is to solve for the equilibrium allocation of information capacity across assets and investors. In our solution, both the number of assets that are being learned about and the mass of investors learning about each asset are determined endogenously. In contrast, previous work assumes that all investors with positive capacity learn about the same asset(s). Since learning about an asset affects the holdings of that asset, the endogenous allocation of investor learning allows us derive rich asset-level predictions, and it is critical to our test of the information mechanism. In equilibrium, learning at the investor level exhibits *specialization*, *preference for volatility*, and *strategic substitutability.* However, learning at the aggregate level exhibits diversification. In particular, we derive a threshold for the aggregate capacity in the economy below which all investors learn only about the most volatile asset. Above this threshold, investors expand their learning towards lower and lower volatility assets.

We provide an analytical characterization of the model's predictions, which we then quantify in the parameterized model. First, in the cross-section of investors, sophisticated investors generate relatively higher capital income due to three forces: (i) they hold larger portfolios of risky assets on average; (ii) they tilt their portfolios towards assets with higher average excess returns; and (iii) they better adjust their portfolios, state by state, towards assets with higher realized excess returns. Of the three forces, the last effect is by far the most important factor in generating capital income differences. Moreover, these forces are amplified by the general equilibrium effect, which pushes unsophisticated investors to reduce their exposure to assets with large sophisticated ownership, due to the impact of sophisticated ownership on prices.

Second, symmetric growth in capacity, interpreted as a general progress in information-processing technologies, disproportionately benefits sophisticated investors. It results in a relative increase in asset ownership by sophisticated investors and an increase in the polarization of capital income. It also generates a robust, unique way in which investors change the composition of their portfolios. Sophisticated investors start with large shares in the most volatile assets, and subsequently continue to expand to lower-volatility assets. At the same time, unsophisticated investors retrench from risky assets and hold safer assets. Growth in aggregate capacity also leads to lower average market returns and higher asset turnover. Overall, these results play an important role in that they cut against plausible alternative explanations for the observed growth in inequality, such as models with heterogeneous risk aversion or differences in trading costs.

To evaluate the quantitative fit of our theoretical predictions to the data, we parameterize the model using U.S. data spanning the period from 1989 to 2012. We use micro-level data on stocks and aggregate retail and institutional portfolios, which allows us to pin down details of the stochastic structure of assets payoffs and information environment. In our parametrization, we set the parameters based on the first half of our sample period, and treat the second sub-period data moments as a testing ground for the dynamic effect coming from general (rather than investor-specific) progress in information technology.

We show that the analytical predictions from the model are qualitatively and quantitatively borne out in the data. First, sophisticated investors, on average, exhibit higher rates of return that are approximately 2.8 percentage points per year higher in the model, compared to a 3 percentage point difference in the data. Second, we show that in response to symmetric growth in technology, sophisticated investors increase their ownership of equities by first entering the most volatile stocks and subsequently moving into stocks with medium and low volatility–a pattern we also document in the data. At the same time, sophisticated investors' entry into equity induces higher asset turnover, in magnitudes consistent with the data, both in the time series and in the cross-section of stocks.

More broadly, our mechanism provides an explanation for the growing presence of sophisticated, institutional investors in risky asset classes, over the last 20-30 years (Gompers and Metrick (2001)). Our mechanism also fits well with a puzzling phenomenon of the last two decades of a growing retrenchment of retail investors from trading and stock market ownership in general (Stambaugh (2014)),[5] even though direct transaction costs, if anything, have fallen significantly. We document such avoidance of risky assets both for direct stock ownership and ownership of intermediated products, such as actively managed equity mutual funds: Direct equity ownership has been falling steadily over the last 30 years, while flows into equity mutual funds coming from less sophisticated, retail investors began their decline

[5]We view the Stambaugh (2014) study as complementary to ours. It aims to explain the decreasing profit margins and activeness of active equity mutual funds using exogenously specified decline in individual investors' stock market participation. In contrast, our study endogenizes such decreasing participation as part of the mechanism which explains income inequality.

and turned negative starting from the early 2000s.

Our paper spans three strands of literature: household finance, rational inattention, and income inequality. While some of our contributions are specific to each individual stream, a unique feature of our work is that we integrate the streams into one unified framework.

Within the household finance literature, the main ideas that we develop build upon an empirical literature on limited capital market participation, growing institutional ownership, household trading decisions, and investor sophistication[6]. While the majority of the studies attribute limited participation rates to differences in market participation costs[7] or preferences, we relate investment decisions to differential access to information across investors.

With respect to the literature on endogenous information choice, our work is broadly related to Sims (1998, 2003). More germane to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013), from which we depart by exploring the role of asset and investor heterogeneity both analytically and quantitatively. Allowing for such non-trivial heterogeneity produces very different implications for portfolio decisions, asset prices, and the evolution of inequality over time.

The literature on income inequality dates back to the seminal work by Kuznets (1953) and has been subsequently advanced by the work of Piketty (2003), Piketty and Saez (2003), Alvaredo, Atkinson, Piketty, and Saez (2013), Autor, Katz, and Kearney (2006), and Atkinson, Piketty, and Saez (2011). In contrast to our paper, a vast majority of that literature focuses on income earned in labor market, and does not relate inequality to heterogeneity in the informational sophistication of investors.

The closest paper in spirit to ours is Arrow (1987), who also considers information differences as an explanation of the income gap. However, his work does not consider endogenous information acquisition and is not a general equilibrium analysis of the economy with het-

[6]For studies on participation, see Mankiw and Zeldes (1991); Ameriks and Zeldes (2001); Gompers and Metrick (2001) illustrate trends in ownership; Barber and Odean (2001), Campbell (2006), Calvet, Campbell, and Sodini (2009b, 2009a), Guiso and Sodini (2012) analyze household trading decisions; Barber and Odean (2000, 2009), Calvet, Campbell, and Sodini (2007), Grinblatt, Keloharju, and Linnainmaa (2012) examine investor sophistication.

[7]See Gomes and Michaelides (2005), Favilukis (2013)

erogeneously informed agents and many assets. Both of these elements are crucial for our results, especially to establish the validity of our mechanism. Another related paper is that of Peress (2004), who examines the role of wealth and decreasing absolute risk aversion in investors' acquisition of information and participation in one risky asset. While some elements of his model are common, his focus is not on capital income inequality. Moreover, we show that heterogeneity across assets and investors is a crucial component to quantitatively capture the evolution of capital income inequality and its underlying economic mechanism.

The rest of the paper proceeds as follows. Section 1 presents the theoretical framework. Section 2 derives analytic predictions, which we subsequently take to the data. Section 3 presents the parametrization of the model and quantifies the information friction. Section 4 establishes our main results about the evolution of capital income inequality, and Section 5 concludes. All proofs and derivations are in the Appendix.

1 Theoretical Framework

A continuum of atomless investors of mass one, indexed by j, solve a sequence of static portfolio choice problems, so as to maximize mean-variance utility over wealth W_j in each period, given common risk aversion coefficient $\rho > 0$. The financial market consists of one risk-free asset, with price normalized to 1 and payoff r, and $n > 1$ risky assets, indexed by i, with prices p_i, and independent payoffs $z_i = \overline{z} + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}\left(0, \sigma_i^2\right)$. The risk-free asset has unlimited supply, and each risky asset has fixed supply, $\overline{x}$. For each risky asset, non-optimizing "noise traders" trade for reasons orthogonal to prices and payoffs (e.g., liquidity, hedging, or life-cycle reasons), such that the net supply available to the (optimizing) investors is $x_i = \overline{x} + \nu_i$, with $\nu_i \sim \mathcal{N}\left(0, \sigma_x^2\right)$, independent of payoffs and across assets.[8]

Prior to making the portfolio decision in each period, investors can choose to obtain information about some or all of the risky assets. Mass $\lambda \in (0,1)$ of investors have high capacity for obtaining information, K_1, and are labeled *sophisticated*, and mass $1-\lambda$ have low capacity, K_2, and are labeled *unsophisticated*, with $0 < K_2 < K_1 < \infty$. Information

[8] For simplicity, we introduce heterogeneity only in the volatility of payoffs, although the model can easily accommodate heterogeneity in supply and in mean payoffs.

is obtained in the form of endogenously designed signals on asset payoffs subject to this capacity limit. The signal choice is modeled following the rational inattention literature (Sims (2003)), using entropy reduction as a measure of the amount of information acquired.

1.1 Investor Optimization

Optimization occurs in two stages. In the first stage, investors solve their information acquisition problem: they choose the distribution of signals to receive in order to maximize expected utility, subject to their information capacity. In the second stage, given the signals they receive, investors update their beliefs about the payoffs and choose their portfolio holdings to maximize utility. We first describe the optimal portfolio choice in the second stage, for a given signal choice. We then solve for the ex-ante optimal signal choice.

Portfolio Choice Given equilibrium prices and posterior beliefs, each investor solves

$$U_j = \max_{\{q_{ji}\}_{i=1}^{n}} E_j \left(W_j\right) - \frac{\rho}{2} V_j \left(W_j\right) \tag{1}$$

$$s.t. \;\; W_j = r\left(W_{0j} - \sum_{i=1}^{n} q_{ji} p_i\right) + \sum_{i=1}^{n} q_{ji} z_i, \tag{2}$$

where E_j and V_j denote the mean and variance conditional on investor j's information set, and W_{0j} is initial wealth. Optimal portfolio holdings are given by

$$q_{ji} = \frac{\widehat{\mu}_{ji} - r p_i}{\rho \widehat{\sigma}_{ji}^2}, \tag{3}$$

where $\widehat{\mu}_{ji}$ and $\widehat{\sigma}_{ji}^2$ are the mean and variance of investor j's posterior beliefs about payoff z_i.

Information Acquisition Choice Each investor can choose to receive a separate signal s_{ji} on each of the asset payoffs, z_i. Given the optimal portfolio choice, ex-ante, each investor chooses the optimal distribution of signals to maximize the ex-ante expected utility, $E_{0j}\left[U_j\right]$. The choice of the vector of signals $s_j = (s_{j1}, ...s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$, is subject to an information capacity constraint, $I\left(z; s_j\right) \leq K_j$, where $I\left(z; s_j\right)$ denotes the

Shannon (1948) mutual information, quantifying the information that the vector of signals conveys about the vector of payoffs. The capacity constraint imposes a limit on the amount of uncertainty reduction that the signals can achieve. Since perfect information requires infinite capacity, each investor faces some residual uncertainty about the realized payoffs.

For analytical tractability, we make the following assumption about the signal structure:

Assumption 1. The signals s_{ji} are independent across assets.

Assumption 1 implies that the total quantity of information obtained by an investor can be expressed as a sum of the quantities of information obtained for each asset.[9] The information constraint becomes $\sum_{i=1}^{n} I\left(z_i; s_{ji}\right) \leq K_j$, where $I\left(z_i; s_{ji}\right)$ measures the information conveyed by the signal s_{ji} about the payoff of asset i.

Investors decompose each payoff into a lower-entropy signal component and a residual component that represents the information lost through this compression: $z_i = s_{ji} + \delta_{ji}$. For tractability, we introduce the following additional assumption:

Assumption 2. The signal s_{ji} is independent of the data loss δ_{ji}.

Since z_i is normally distributed, Assumption 2 implies that s_{ji} and δ_{ji} are also normally distributed. By Cramer's Theorem, $s_{ji} \sim \mathcal{N}\left(\overline{z}, \sigma_{sji}^2\right)$ and $\delta_{ji} \sim \mathcal{N}\left(0, \sigma_{\delta ji}^2\right)$ with $\sigma_i^2 = \sigma_{sji}^2 + \sigma_{\delta ji}^2$.[10] Hence, posterior beliefs are normally distributed random variables, independent across assets, with mean $\widehat{\mu}_{ji} = s_{ji}$ and variance $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$. Intuitively, a perfectly precise signal results in no information loss, such that posterior uncertainty is zero. Conversely, a signal that consumes no information capacity discards all information about the realized payoff, returning only the mean payoff, $\overline{z}$, and leaving an investor's posterior uncertainty equal to her prior uncertainty.

Using this signal structure and the resulting distribution of expected excess returns, the

[9] Assumption 1 is common in the literature. Allowing for potentially correlated signals requires a numerical approach, and is beyond the scope of this paper.

[10] In general, the optimal signal structure may require correlation between the signal and the data loss, but Assumption 2 maintains analytical tractability.

investor's information problem becomes choosing the variance of posterior beliefs to solve[11]

$$\max_{\{\widehat{\sigma}_{ji}^2\}_{i=1}^n} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \quad s.t. \quad \prod_{i=1}^n \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}, \tag{4}$$

where G_i represents the equilibrium utility gain from learning about asset i. This gain represents the shadow value of investing capacity in asset i in equilibrium. It is a function of the distribution of expected excess returns only, and hence is common across investor types and taken as given by each investor.

Lemma 1. *The solution to the maximization problem (4) is a corner: each investor allocates her entire capacity to learning about a single asset from the set of assets with maximal utility gains. The posterior beliefs of an investor j, learning about asset $l_j \in \arg\max_i G_i$, are normally distributed, with mean and variance given by*

$$\widehat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j \end{cases} \quad \text{and} \quad \widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \tag{5}$$

Conditional on the realized payoff z_i, the signal is normally distributed with mean $E\left(s_{ji}|z_i\right) = \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i$, and variance $V\left(s_{ji}|z_i\right) = \left(1 - e^{-2K_j}\right)e^{-2K_j}\sigma_i^2$.

The linear objective function and the convex constraint imply that each investor specializes, learning about a single asset. She always picks an asset with the highest gain G_i and hence all assets that are learned about in equilibrium will have the same gains. Which assets these are is endogenously determined in equilibrium, which we characterize below.

1.2 Equilibrium

Equilibrium Prices Given the solution to each investor's portfolio and information problem, market clearing pins down equilibrium prices as linear combinations of the shocks.

[11]The investor's objective omits terms that do not affect the optimization. For detailed derivations, see the Appendix.

Lemma 2. *The price of asset i is given by $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$, with*

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{(1+\Phi_i)}\right], \quad b_i = \frac{\Phi_i}{r\,(1+\Phi_i)}, \quad c_i = \frac{\rho\sigma_i^2}{r\,(1+\Phi_i)}, \tag{6}$$

where $\Phi_i \equiv m_{1i}\left(e^{2K_1}-1\right)+m_{2i}\left(e^{2K_2}-1\right)$ is a measure of the information capacity allocated to learning about asset i in equilibrium, $m_{1i} \in [0,\lambda]$ is the mass of sophisticated investors who choose to learn about asset i, and $m_{2i} \in [0, 1-\lambda]$ is the mass of unsophisticated investors who choose to learn about asset i, with $\sum_{i=1}^{n} m_{1i} = \lambda$ and $\sum_{i=1}^{n} m_{2i} = 1-\lambda$.

The price of an asset reflects the asset's payoff and effective supply shocks, with relative importance determined by mass of investors learning about the asset through Φ_i, which is a measure of the total capacity that the market allocates to learning about asset i in equilibrium. If there is no information capacity ($K_1 = K_2 = 0$), or for assets that are not learned about ($m_{1i} = m_{2i} = 0$), the price only reflects the noise trader shock ν_i. As the capacity allocated to an asset increases, the asset's price co-moves more strongly with the underlying payoff (c_i decreases and b_i increases, though at a decreasing rate). In the limit, as $K_j \to \infty$, the price approaches the discounted realized payoff, z_i/r, and noise traders become irrelevant for price determination.

Equilibrium Learning Using equilibrium prices, we determine the assets that are learned about and the mass of investors learning about each asset. Without loss of generality, let assets be ordered such that $\sigma_i > \sigma_{i+1}$ for all $i \in \{1, ..., n-1\}$. Let $\xi_i \equiv \sigma_i^2\left(\sigma_x^2 + \overline{x}^2\right)$ summarize the properties of asset i. Then the gain from learning about asset i is

$$G_i = \frac{1+\rho^2\xi_i}{\left(1+\Phi_i\right)^2}. \tag{7}$$

Lemma 3. *The allocation of information capacity across assets, $\{\Phi_i\}_{i=1}^{n}$, is uniquely pinned* *down by:*

$$
\begin{aligned}
G_i &= \max_{h \in \{1,...,n\}} G_h, && \forall i \in \{1, ..., k\}, && (8)\\
G_i &< \max_{h \in \{1,...,n\}} G_h, && \forall i \in \{k+1, ..., n\}, && (9)\\
\sum_{i=1}^{k} \Phi_i &= \phi, && && (10)
\end{aligned}
$$

where k denotes the endogenous number of assets with strictly positive learning mass in equilibrium, and $\phi \equiv \lambda \left(e^{2K_1} - 1\right) + (1-\lambda)\left(e^{2K_2} - 1\right)$ is a measure of the total capacity for processing information available in the economy.

In a symmetric equilibrium in which $m_{1i} = \lambda m_i$ and $m_{2i} = (1-\lambda) m_i$, where m_i is the total mass of investors learning about asset i, the masses $\{m_i\}_{i=1}^{n}$ are given by

$$
\begin{aligned}
m_i &= \frac{c_{i1}}{C_k} + \frac{1}{\phi}\left(\frac{k c_{i1}}{C_k} - 1\right), && \forall i \in \{1, ..., k\}, && (11)\\
m_i &= 0, && \forall i \in \{k+1, ..., n\}, && (12)
\end{aligned}
$$

where $c_{i1} \equiv \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_1}} \leq 1$, with equality iff *$i = 1$, and $C_k \equiv \sum_{i=1}^{k} c_{i1}$.*

The model uniquely pins down the total capacity allocated to each asset, Φ_i, but it does not separately pin down m_{1i} and m_{2i}. Since the asset-specific gain from learning is the same for both types of investors, we assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population. In turn, this implies a unique set of masses $\{m_i\}_{i=1}^{n}$, with $\Phi_i = \phi m_i$. Lemma 3 implies the following three properties:

$$
\frac{\partial G_i}{\partial \sigma_i^2} > 0, \quad \frac{\partial G_i}{\partial m_i} < 0, \quad \frac{\partial G_i}{\partial \phi} \leq 0.
$$

Learning in the model exhibits preference for volatility (high σ_i^2) and strategic substitutability (low m_i). Furthermore, the value of learning about an asset also falls with the aggregate amount of information in the market (ϕ), since higher capacity overall increases the comovement between prices and payoffs, thereby reducing expected excess returns.

For sufficiently low information capacity, all investors learn only about the most volatile asset: for $\phi \in (0, \phi_1]$, $m_1 = 1$ and $m_i = 0$ for all $i > 1$, where

$$\phi_1 \equiv \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_2}} - 1. \tag{13}$$

This threshold endogenizes single-asset learning as an optimal outcome for low enough information capacity relative to asset dispersion. As the overall capacity in the economy increases above this threshold, investors expand their learning towards lower volatility assets. We define the thresholds for learning as follows:

Definition 1. Let ϕ_k be such that for any $\phi \leq \phi_k$, at most the first k assets are actively traded (learned about) in equilibrium, while for $\phi > \phi_k$, at least the first $k+1$ assets are actively traded in equilibrium.

Lemma 3 implies that the threshold values of aggregate information capacity are monotonic: $0 < \phi_1 < \phi_2 < ... < \phi_{n-1}$. For sufficiently high information capacity, or alternatively, for low enough dispersion in assets volatilities, all assets are actively traded, thus endogenizing the assumption employed in models with exogenous signals.

In the presence of assets heterogeneity, even if many assets are learned about, there is heterogeneity in the information capacity allocated to each of the actively traded assets. Since the equilibrium gain is increasing in volatility and decreasing in m_i, the mass of investors learning about each asset is increasing in volatility. In turn, this heterogeneity has implications for holdings, returns, and turnover in the cross-section of assets. Additionally, if we let the degree of dispersion in asset payoff volatilities vary, learning will also vary, with periods with high dispersion being characterized by more concentrated learning, and periods with low dispersion characterized by more diversified learning (and hence portfolios).

We next characterize learning in response to variation in the level of investor capacities.

Lemma 4. *Let $\phi \in (\phi_{k-1}, \phi_k]$ such that $k > 1$ assets are actively traded, and consider an increase in ϕ such that $k' \geq k$ is the new equilibrium number of actively traded assets.*

(i) There exists a threshold asset $\bar{\imath} < k'$, such that m_i is decreasing in ϕ for all assets $i \in \{1, ..., \bar{\imath}\}$, and increasing in ϕ for all assets $i \in \{\bar{\imath} + 1, ..., k'\}$.

(ii) The quantity (ϕm_i) is increasing in ϕ for all assets $i \in \{1, ..., k'\}$.

(iii) For an increase in ϕ generated by a symmetric growth, $K_j' = (1 + \gamma) K_j$, with $\gamma \in (0, 1)$, the quantity $m_i(e^{2K_j} - 1)$, $j \in \{1, 2\}$, is increasing in K_j at an increasing rate, for assets $i \in \{\bar{\imath} + 1, ..., k'\}$. For assets $i \in \{1, ..., \bar{\imath}\}$, $m_i \left(e^{2K_1} - 1\right)$ grows while $m_i \left(e^{2K_2} - 1\right)$ grows by less, or even falls if capacity dispersion is large enough.

Lemma 4 shows the diversification effect. First, as the amount of aggregate capacity ϕ increases, the mass of investors learning about the most volatile assets decreases as some investors shift to learning about less volatile assets. Nevertheless, the total amount of capacity allocated to each asset (ϕm_i) strictly increases for all assets that are actively traded. Lastly, *symmetric* growth in capacity benefits the sophisticated group disproportionately: this group allocates more capacity to each asset relative to the unsophisticated group, which in turn generates asymmetry in investment patters. In Section 2, we use these results to derive analytic predictions on the patterns of investment in response to changes in capacity.

2 Analytic Results

In this section, we present a set of analytic results implied by our information friction, and we discuss how variations in the baseline framework affect these results.

2.1 Model Predictions

Heterogeneous Capacity Our first set of analytic results show that heterogeneity in information capacity across investors drives capital income inequality in the cross-section, through differences in average portfolio holdings and through heterogeneity in the ability to adjust holdings to shocks. Let q_{1i} and q_{2i} denote the average per-capita holdings of asset i for sophisticated and unsophisticated investors, respectively. The per-capita asset-level holdings of sophisticated investors are

$$q_{1i} = \left(\frac{\overline{z}_i - rp_i}{\rho \sigma_i^2} \right) + m_i \left(e^{2K_1} - 1 \right) \left(\frac{z_i - rp_i}{\rho \sigma_i^2} \right), \tag{14}$$

and those of the relatively unsophisticated investors are defined analogously. Per-capita holdings are a weighted average of the quantity that would be held under the investors' prior beliefs and a quantity that is increasing in the realized excess return, scaled by an asset-specific term that captures the amount of information capacity allocated to this asset by this investor group. For actively traded assets, heterogeneity in capacities generates differences in ownership across investor types at the asset level:

$$q_{1i} - q_{2i} = m_i \left(e^{2K_1} - e^{2K_2} \right) \left(\frac{z_i - rp_i}{\rho \sigma_i^2} \right). \tag{15}$$

Integrating over the realizations of the state (z_i, x_i), the expected per-capita ownership difference, as a share of the supply of each asset, is also asset specific,

$$\frac{E\left[q_{1i} - q_{2i}\right]}{\overline{x}} = \left(e^{2K_1} - e^{2K_2} \right) \frac{m_i}{1 + \phi m_i}, \tag{16}$$

which implies that the portfolio of the unsophisticated investor is not simply a scaled down version of the sophisticated portfolio. Rather, the portfolio weights *within* the class of risky assets are also different for the two investor types.

Proposition 1 (**Ownership**). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*
(i) $E\left[q_{1i} - q_{2i}\right] / \overline{x} > 0$;
(ii) $E\left[q_{1i} - q_{2i}\right] / \overline{x}$ is increasing in $E\left[z_i - rp_i\right]$;
(iii) $q_{1i} - q_{2i}$ is increasing in $z_i - rp_i$.

The average sophisticated investor (*i*) holds a larger portfolio of risky assets on average, (*ii*) tilts her portfolio towards assets with higher expected excess returns, and (*iii*) adjusts ownership, state by state, towards assets with higher *realized* excess returns.

These results imply that sophisticated investors generate relatively higher capital income, asset by asset, both on average and state by state. Let π_{1i} and π_{2i} denote the capital income per capita from trading asset i, for sophisticated and unsophisticated investors, respectively, with $\pi_{1i} \equiv q_{1i}(z_i - rp_i)$ and $\pi_{2i} \equiv q_{2i}(z_i - rp_i)$. For actively traded assets, heterogeneity in ownership generates heterogeneity in capital income across investor types at the asset level:

$$\pi_{1i} - \pi_{2i} = m_i \left(e^{2K_1} - e^{2K_2}\right) \frac{(z_i - rp_i)^2}{\rho \sigma_i^2}. \tag{17}$$

Integrating over the realizations of the state (z_i, x_i), the average capital income difference is

$$E[\pi_{1i} - \pi_{2i}] = \frac{1}{\rho} m_i \left(e^{2K_1} - e^{2K_2}\right) G_i, \tag{18}$$

where G_i is the gain from learning about asset i.

Proposition 2 (**Capital Income**). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*
(i) $\pi_{1i} - \pi_{2i} \geq 0$, with strict inequality in states with non-zero realized excess returns;
(ii) $E[\pi_{1i} - \pi_{2i}]$ is increasing in asset volatility σ_i.

The average sophisticated investor realizes larger profits in states with positive excess returns, and incurs smaller losses in states with negative excess returns, because her holdings, q_{1i}, co-move more strongly with the realized state, $z_i - rp_i$. Moreover, the biggest difference in profits, on average, comes from investment in the more volatile, higher expected excess return assets.

The differential adjustment to shocks also implies differences in trading intensity, which provides an additional set of testable implications. Formally, we define the expected volume of trade in asset i as $V_i \equiv \int |q_{ji,t} - q_{ji,t-1}| \, dj$. We can decompose total volume into parts coming from four different investor groups, by their level of sophistication and by whether or not they are learning about the asset:[12]

$$V_i = \lambda m_i V_{1i} + (1-\lambda) m_i V_{2i} + \lambda (1-m_i) V_{3i} + (1-\lambda)(1-m_i) V_{4i},$$

where V_{1i} is the expected per capita volume of sophisticated investors actively trading asset i, whose mass in the population is λm_i; the remaining terms are analogous. For each group g, volume is proportional to the group's cross-sectional standard deviation of holdings, $V_{gi} = \frac{2\sigma_{gi}}{\sqrt{\pi}}$. Hence, the average turnover of asset i, $T_i \equiv V_i/\overline{x}$, is given by

$$T_i = \frac{2m_i}{\rho \overline{x} \sigma_i \sqrt{\pi}} \left[\lambda \sqrt{e^{2K_1} - 1} + (1-\lambda) \sqrt{e^{2K_2} - 1} \right]. \tag{19}$$

Proposition 3 (**Turnover**). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then,*
(i) For $i \in \{1, ..., k\}$, average turnover by investor group satisfies $T_{1i} > T_{2i} > T_{3i} = T_{4i} = 0$.
(ii) For $h \in \{k+1, ..., n\}$, average turnover is $T_h = 0$.

Hence, sophisticated investors generate more asset turnover, since having higher capacity to process information enables them to take larger and more volatile positions, relative to unsophisticated investors. Moreover, assets that are actively traded, in turn, have a higher turnover compared with assets that are passively traded (based only on prior beliefs). In fact, for passively traded assets, average turnover is zero.

Larger Capacity Dispersion Our second set of analytic results show that increased dispersion in capacities implies further polarization in holdings, which in turn leads to a growing capital income polarization. Intuitively, greater dispersion in information capacity implies that sophisticated investors receive relatively higher-quality signals about the fundamental payoffs, which enables them to respond more strongly to realized state.

Proposition 4 (**Capacity Dispersion**). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Consider an increase in capacity*

[12]The average volume of noise traders is zero. Among the optimizing investors, we assume that investors do not change groups over time.

dispersion of the form $K_1' = K_1 + \Delta_1 > K_1$*,* $K_2' = K_2 - \Delta_2 < K_2$*, with* Δ_1 *and* Δ_2 *chosen such that the total information capacity* ϕ *remains unchanged. Then, for* $i \in \{1, ..., k\}$*,*

(i) Asset prices and excess returns remain unchanged.

(ii) The difference in ownership shares $(q_{1i} - q_{2i})/\overline{x}$ *increases.*

(iii) Capital income becomes more polarized as the ratio π_{1i}/π_{2i} *increases state by state.*

Increasing the level of capacity dispersion while leaving the aggregate measure of information in the economy unchanged, does not affect equilibrium prices, since keeping ϕ unchanged implies that both the number of assets learned about and the mass of investors learning about each asset remain unchanged. Hence the adjustment reflects a pure transfer of ownership from the relatively unsophisticated investors (who now have even lower capacity) to the more sophisticated investors (who now have even higher capacity). This reallocation of holdings leads to higher capital income inequality without any general equilibrium effects.

Symmetric Capacity Growth Our third and most important set of analytic results shows that in the presence of initial heterogeneity, technological progress in the form of *symmetric* growth in information capacity leads to a disproportionate increase in ownership of risky assets by sophisticated investors, and to growing capital income polarization. Symmetric growth is modeled as a common growth rate of both K_1 and K_2,

Proposition 5 (**Symmetric Growth**)**.** *Let* $K_1 > K_2$ *and* $\phi_{k-1} \leq \phi < \phi_k$*, such that the first* $k > 1$ *assets are actively traded in equilibrium. Consider an increase in* ϕ *generated by a symmetric growth in capacities to* $K_1' = (1+\gamma)K_1$ *and* $K_2' = (1+\gamma)K_2$*,* $\gamma \in (0,1)$*. Let* $k' \geq k$ *denote the new equilibrium number of actively traded assets. Then, for* $i \in \{1, ..., k'\}$*,*

(i) Average asset prices increase and average excess returns decrease.

(ii) The average ownership share of sophisticated investors $E[q_{1i}]/\overline{x}$ *increases and the average ownership share of unsophisticated investors* $E[q_{2i}]/\overline{x}$ *decreases.*

(iii) Average capital income becomes more polarized, as the ratio $E[\pi_{1i}]/E[\pi_{2i}]$ *increases.*

(iv) Total market turnover $T \equiv \sum_i V_i/n\overline{x}$ *increases.*

First, higher capacity for processing information means that investors receive more accurate signals about the realized payoffs. Hence, their demand for assets co-moves more closely with the realized state, which implies that prices contain a larger amount of information about the fundamental shocks. As a result, the equilibrium implies lower average returns, larger and more volatile positions, and higher market turnover.

Second, a symmetric growth in capacity that benefits both sophisticated and unsophisticated investors has two effects on portfolio holdings and capital income inequality: a partial equilibrium effect and a general equilibrium effect. Absent any equilibrium price adjustment, the average holdings of risky assets and the comovement between holdings and the realized state increase for both investor types. However, because growth in capacity benefits investors who already have relatively high capacity, the benefits accrue more for sophisticated investors. Further, in contrast to the case of increased dispersion, a symmetric change in information capacity affects equilibrium prices. As sophisticated investors increase their demand for risky assets, this drives up average prices, reducing the expected profits of unsophisticated investors, who in turn reduce their average holdings of risky securities.

2.2 The Value of Prices

In our analysis so far, we have presented the information acquisition problem in terms of a constraint on information obtained through private signals alone, excluding the information contained in prices. When some investors acquire information through private signals, prices become informative about asset payoffs, because they reflect the demand of these privately informed investors. In the literature on portfolio choice with exogenous signals, investors are often assumed to learn about payoffs not only from their private signals, but also from equilibrium prices, which aggregate the information of all investors in the market (e.g., Admati (1985)). Would investors with an endogenous signal choice have an incentive to allocate any capacity to learning from prices? We show that if the information contained in prices is costly to process, then prices are an inferior source of information compared with private signals.

We consider the signal choice of an individual investor, taking the choices of all other investors as given by the equilibrium obtained in Section 1.2. Processing information through either prices or private signals consumes the investor's capacity. Hence, whatever the source of information, the investor cannot acquire a total quantity beyond her capacity K_j.

Proposition 6 (**Prices**). *If learning about prices consumes capacity, then the capacity-constrained investor chooses to devote all her capacity to learning about payoffs through private signals on asset payoffs, rather than devoting any capacity to learning from prices.*

Intuitively, prices represent an indirect way of learning about asset payoffs, which are ultimately what investors seek to learn. Our proof follows the logic of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013), although it is derived for a different information structure and extended to include the case in which the information content of prices is not processed perfectly.

If processing the noise trader shock also consumes capacity, then Proposition 6 implies that investors will not allocate any capacity to learning about the supply shock, ν_i. Learning about the activity of noise traders is not useful *unless* that information is combined with information processed from prices. It is only the joint information on both variables that informs investors about payoffs.

2.3 Alternative Specifications

Free prices To assess the sensitivity of our model to the assumption that prices consume capacity, we also solve the model under the assumption that processing the information content of prices is costless. We consider a setting in which investors can design a signal structure that conditions the private signal choice on the price realization (just as the choice of asset holdings conditions on the price realization). Signal acquisition is subject to a modified information constraint, $I(s_j; z|p) \leq K_j$, where $I(s_j; z|p)$ denotes the conditional relative entropy, measuring the information about z conveyed by private signals given prices.[13] This change partially erodes the informational advantage of sophisticated investors. As a result, the heterogeneity in both holdings and returns is reduced (though not eliminated). Nonetheless, our results pertaining to the preference for volatility in learning and the expansion of

[13]This formulation provides an upper bound on the information value of prices.

learning due to aggregate capacity growth continue to hold.[14]

Additive constraint Changing the form of the information constraint does not affect equilibrium outcomes qualitatively. Suppose we replaced the entropy constraint with a constraint on the sum of the ratios of variances of prior and posterior beliefs, similar to Grossman and Stiglitz (1980) and Van Nieuwerburgh and Veldkamp (2009): $\sum_{i=1}^{n}\left(\frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2}-1\right)\leq K_j$, while maintaining the same signal structure. Maximization continues to imply that each investor specializes, learning about a single asset. Moreover, investors choose to learn about the same assets as in the baseline specification. Heterogeneity in capacities continues to generate heterogeneity in both holdings and returns. However, dynamically, in response to symmetric growth in capacity, capital income inequality grows at a slower rate, since the additive constraint reduces the marginal benefit of additional capacity for sophisticated investors relative to the benefit for unsophisticated investors.

Additive noise signals Previous work on information choice typically assumes an *additive noise* signal structure, $\widetilde{s}_{ji}=z_i+\widetilde{\delta}_{ji}$. In our setup, additive noise signals yield exactly the same equilibrium outcomes as the compressed signals we employ, given the assumptions that we have made in setting up the signal structure. However, given our use of an entropy constraint, additive noise poses some interpretation challenges. In our specification, agents compress the state into a simpler signal, with the residual representing information about the state that is lost due to the processing constraint. Hence, no learning amounts to the simplest possible signal, equal to the mean payoff in all states. As agents devote more and more capacity to learning about the state, they *lose* less and less information about the state (rather than *adding* less and less noise to it). Conversely, in the additive noise framework, a lower capacity constraint amounts to adding more noise to the realized state, and no learning amounts to adding infinite noise to the realized state. Hence, we find the additive noise signal structure more appealing for applications in which agents receive signals exogenously, rather than for settings in which they design the signals themselves. Finally,

[14]The derivations for this and the subsequent alternative specifications follow the baseline derivation in the Appendix and are available upon request.

although mathematically equivalent here, in general, the two formulations need not give the same results (Matějka (2011), Stevens (2014)).

Risk Aversion Heterogeneity Capital income inequality can be also driven by differences in risk aversion among investors, in the absence of any heterogeneity in the capacity to process information about asset payoffs. In particular, if one group of investors were less risk averse they would hold a greater share of risky assets, and hence they would have higher expected capital income.[15] Within our mean-variance specification, a growing difference in risk aversion produces growing *aggregate* ownership in risky assets of less risk averse investors, and a uniform, proportional retrenchment from risky assets of more risk averse investors. However, it does not generate *(i)* differences in portfolio weights within a class of risky assets, *(ii)* investor-specific rates of return on equity, or *(iii)* differential growth in ownership by asset volatility.[16]

3 Quantifying the Information Friction

In this section, we first parameterize the model using stock-level micro data by asset class and investor type. Then, we evaluate the quantitative power of the proposed information friction vis-a-vis investment patterns in the data. Finally, we present results on the dynamics of heterogeneity in returns, participation, and portfolio composition that help us identify our economic mechanism in the data.

3.1 Parametrization

Our analytical design combines a portfolio framework with information frictions. Thus, in order to parameterize the model it is essential that we use data with a similar level of granularity. To this end, we use institutional portfolio holdings from a Thomson Reuters

[15]Such setting would also encompass situations in which investors are exposed to different levels of volatility in areas outside capital markets, like labor income.

[16]In a CRRA model, portfolio weights would also be identical *across risky assets*; hence, even in that specification, rates of return on equity would be equalized across investor types.

dataset, which contain a large sample of portfolios of publicly traded equity held by institutional investors, and comes from quarterly reports required by law and submitted by institutional investors to the Securities and Exchange Commission.[17]

Investor Types To map the model to the data, we study portfolios of investors with different degrees of sophistication. Sophisticated investors are defined as investment companies or independent advisors (types 3 and 4) in the Thomson data set. These investors include wealthy individuals, mutual funds, and hedge funds. Among all types, these groups are particularly active in their information production efforts; in turn, other groups, such as banks, insurance companies, or endowments and pensions are more passive by nature. Our definition of unsophisticated investors is other shareholders who are not part of Thomson data. These include individual (retail) investors.

To provide the empirical verification of the proposed investor classification, we show the evolution of the cumulative returns of portfolios held by the two types of investors over the period 1989-2012. We proceed in three steps. First, we obtain the market value of each stock held by all investors of a given type. The market value of each stock is the product of the number of combined shares held by a given investor type and the price per share of that stock, obtained from CRSP. Since the number of shares held by unsophisticated investors is not directly observable, we impute this value by taking the difference between the total number of shares available for trade and the number of shares held by all institutional investors. Second, we calculate the value shares of each stock in the aggregate portfolio by taking the ratio of market value of each stock relative to the total value of the portfolio of each investor type. Third, we obtain the return on the aggregate portfolio by matching each asset share with their next month realized return and calculating the value-weighted aggregated return. We repeat this procedure separately for sophisticated and unsophisticated investors. Figure 1 shows the cumulative values of $1 invested by each group in January 1989, using the aggregated monthly returns through December 2012.

[17] While the official requirement for reporting is that the minimum asset size exceed 100 million, such that not all investors are in the data; in reality, the data are comprehensive, as more than 95% of all dollar investments are reported.

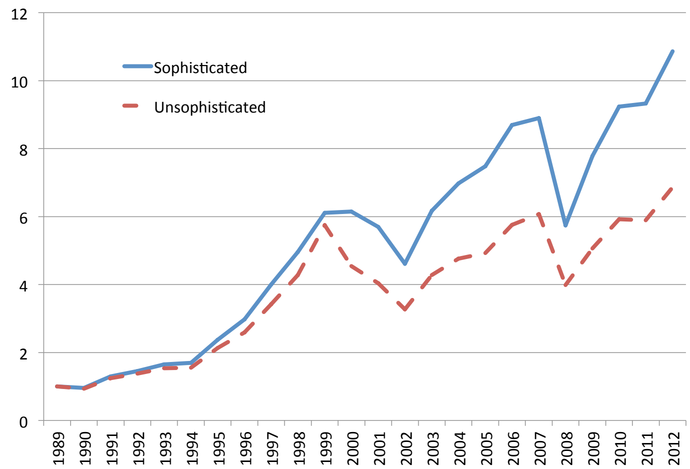


Figure 1: Cumulative Portfolio Returns by Investor Type.

Our results indicate that the portfolios of sophisticated investors systematically outperform those of unsophisticated investors. The value of \$1 invested in January 1989 grows to \$5.32 at the end of 2012 for sophisticated investors versus \$3.28 for unsophisticated investors. Hence, our investor classification implies superior investment strategies of the investor group we label as sophisticated.

Empirical Targets We parameterize the model by targeting statistics based on stock market data. Table 1 presents the complete parametrization. For parsimony, we restrict some parameters and normalize the natural candidates. In particular, we normalize the mean payoff to $\bar{z}_i = 10$ and the asset supply to $\bar{x}_i = 5$ for all assets, we restrict the volatilities of the noise shocks, $\sigma_{xi} = \sigma_x$ for all assets, and we set the number of assets to $n = 10$. The remaining parameters are the information capacities of the two investor types ($K_1 = 0.598$ and $K_2 = 0.0598$), the fraction of sophisticated investors in the population ($\lambda = 0.2$), the risk-free interest rate ($r = 2.5\%$), the risk aversion parameter ($\rho = 1.12$), the volatility of the noise shock ($\sigma_x = 0.41$), and the volatilities of the payoffs (σ_i), for which we normalize the lowest volatility, $\sigma_n = 1$, and assume that volatility changes linearly across assets. Specifically, we set the slope of the volatility line to $\alpha = 0.53$ and set $\sigma_i = \sigma_n + \alpha(n - i)/n$, which implies that volatilities range from $\sigma_n = 1$ to $\sigma_1 = 1.48$.

These parameter values are chosen to jointly match key moments from stock-level micro

Table 1: Parameter Values

Parameter	Symbol	Value	Target
Mean payoff, supply	$\bar{z}_i$, $\bar{x}_i$	10, 5 for all i	Normalization
Number of assets	n	10	Normalization
Risk-free rate	r	2.5%	3-month T-bill $-$ inflation $=$ 2.5%
Risk aversion	ρ	1.12	Market return $=$ 11.9%
Vol. of noise shocks	σ_{xi}	0.41 for all i	Average turnover $=$ 9.7%
Vol. of asset payoffs	σ_i	$\in [1, 1.48]$	p90/p50 of idio return vol $=$ 3.54
Information capacities, fraction sophisticated	K_1, K_2, λ	0.598, 0.0598, 0.2	Sophisticated share $=$ 23%
			Share actively traded $=$ 50%
			Ratio of K_1 to K_2 $=$ 10%

data and aggregate investor-type equity shares for the first half of our sample, 1989-2000. We take this sub-sample as the point of departure for our dynamic comparative statics exercises. We match the following targets: *(i)* the equity ownership share of sophisticated investors of 23%; *(ii)* the average return on 3-month Treasury bills minus the inflation rate, equal to 2.5%; *(iii)* the average annualized stock market return in excess of the risk-free rate, equal to 11.9%; *(iv)* the average monthly equity turnover (defined as the total monthly volume divided by the number of shares outstanding), equal to 9.7%; *(v)* the ratio of the 90^{th} percentile to the median of the cross-sectional idiosyncratic volatility of stock returns, equal to 3.54; and *(vi)* the fraction of assets that investors learn about, which, in the absence of empirical guidance, we arbitrarily set to 50%. This procedure leaves us with one key parameter: the relative information capacity of sophisticated to unsophisticated investors, K_1/K_2. In this section, we set this parameter to 10%, while in Section 4, we identify this parameter using data from the Survey of Consumer Finances.

3.2 Return Differences

In this section, we evaluate the quantitative power of our information friction by contrasting the implied return differential with the stock-level micro evidence. We report the results in Table 2. The parameterized model implies a 2.8 percentage point advantage in the average portfolio returns of the sophisticated investors (who earn an average return of 14.6%) relative to the unsophisticated investors (who earn an average return of 11.8%). This difference is comparable to the 3.0 percentage point difference in the data for the 1989-2000 period (with average portfolio returns of 13.4% versus 10.4%). Thus, the model can generate the empirical difference in returns, while matching other aggregate targets.

Table 2: Average Portfolio Returns: Data and Model

	1989-2000	
Portfolio Return	Data	Model
Sophisticated investors	13.4%	14.6%
Unsophisticated investors	10.4%	11.8%
Unsophisticated investors + Noise traders		11.2%

In addition to the optimizing sophisticated and unsophisticated investors, the model features a third type of agent: noise traders, who trade for reasons unrelated to asset payoffs and prices. The question is whether to classify them as sophisticated or unsophisticated. On the one hand, such traders generate losses from their trading activity, and their trading strategies seem inefficient. On the other hand, sophisticated investors may also occasionally face shocks that force them to trade assets for reasons other than the price or payoff, for hedging or liquidity reasons. For our parametrization, these considerations are not quantitatively important. The last row of Table 2 reports the portfolio returns of a joint unsophisticated plus noise trader portfolio. The joint portfolio generates a return that is quantitatively close to that of the pure unsophisticated portfolio (11.2% versus 11.8%). Since noise trader demand is a mean zero random variable with relatively small volatility, the allocation of noise traders to either investor group has quantitatively small effects.

Return Decomposition As our analytical results suggest, sophisticated investors outperform unsophisticated investors for two reasons (summarized in Propositions 1 and 2): *(i)* they are more exposed to risk because they hold a larger share of risky assets (compensation for risk); and *(ii)* they have informational advantage (compensation for skill). In order to shed light on the relative importance of these two effects, we decompose the returns of each investor type by computing the unconditional expectation of the return on the portfolio held by investor type $j \in \{S, U\}$:

$$R_j = E \sum_i \omega_{jit}(r_{it} - r) = \sum_i Cov(\omega_{jit}, r_{it}) + \sum_i E\omega_{jit} E[r_{it} - r], \tag{20}$$

where $r_{it} = z_{it}/p_{it}$ is the time t return on asset i and ω_{jit} is the portfolio weight of asset i for investor j at time t as $\omega_{jit} = q_{jit}p_{it}/\sum_l q_{jlt}p_{lt}$. The first term of the decomposition captures the covariance conditional on investor j information set, i.e. the investor's reaction to information flow via portfolio weight adjustment (*skill effect*); the second term captures the *average effect*, unrelated to active trading.

Quantitatively, the skill effect accounts for the majority of the return differential in the model. To show that, we compute the counterfactual return of sophisticated investors if their skill effect were the same as that of unsophisticated investors, but their average holdings were still the same

$$\hat{R}_S = \sum_i Cov(\omega_{Uit}, r_{it}) + \sum_i E\omega_{Sit} E[r_{it} - r]. \tag{21}$$

Such a portfolio would generate an annualized return of 12.4%, which implies that the compensation for skill accounts for approximately 80% of the 2.8% return differential between the sophisticated and the unsophisticated portfolios.

3.3 Testing the Mechanism

In this section, we generate a set of dynamic predictions of the model and compare them to the corresponding data moments in order to provide support for our mechanism. These are robust predictions of our mechanism and are proven analytically in Section 2. Below, we show a good fit of these results not only qualitatively but also quantitatively.

To test our mechanism, we explore the consequences of a symmetric change in capacities of both investor types, targeting the change in the equity ownership share of sophisticated investors. In the data, this share grew to an average of 46% in the 2001-2012 period, from an average of 23% in the 1989-2000 period. We find that the progress in information capacity required to achieve this target amounts to an annual growth of 9.7% (for 11 years, from the middle of the first sub period to the middle of the second sub period).[18] Hence, in the presence of initial capacity dispersion, subsequent symmetric capacity growth is sufficient to generate a disproportionate growth in sophisticated ownership, and retrenchment of unsophisticated investors from risky assets.

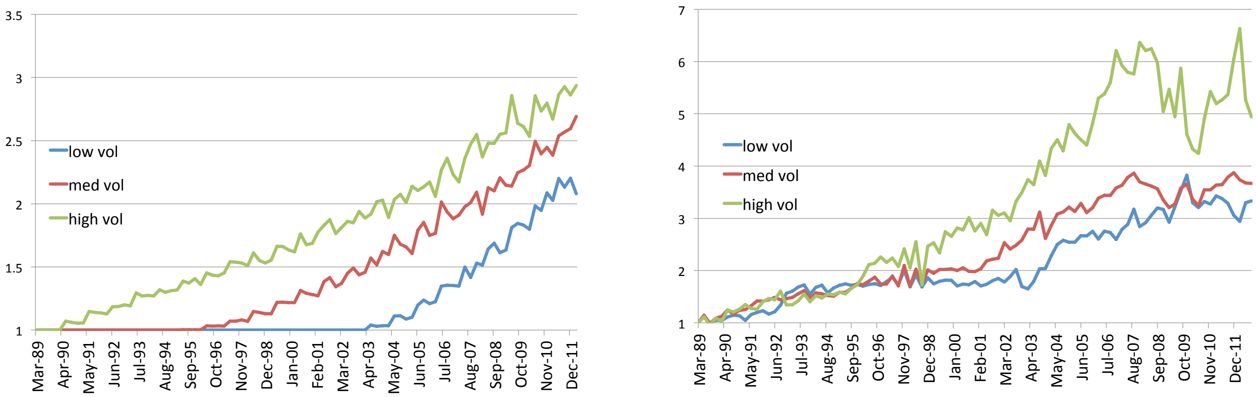
Market Averages In the model, symmetric growth in information capacities implies large changes in average market returns, cross-sectional return differentials, and turnover (Proposition 5). Table 3 reports the model predictions and their empirical counterparts.

Table 3: Market Averages: Data and Model

	2001-2012	
Statistic	Data	Model
Market Returns	2.4%	3.1%
Sophisticated portfolio	2.9%	3.3%
Unsophisticated portfolio	1.6%	3.0%
Unsophisticated + Noise traders portfolio	-	2.95%
Average Equity Turnover	16.0%	14.7%
Sophisticated Ownership Share (target)	46.0%	46.0%

Both the model and the data exhibit a decrease in market return and in the return differential between sophisticated and unsophisticated portfolios. The lower market return is a result of an increase in the quantity of information, as prices track payoffs more closely than in the initial sample period, implying lower excess returns. The model also predicts a sharp increase in average asset turnover, in magnitudes consistent with the data. As with the market return, this result is a direct implication of our mechanism and is not driven by changes in fundamental asset volatilities, which remain unchanged. Intuitively, higher turnover is driven by more informed trading by sophisticated investors, due to their holding a larger share of the market and receiving more precise signals about asset payoffs

[18] This growth results in final capacities of $K_1 = 1.654$ and $K_2 = 0.1654$.



(a) (b)

Figure 2: Cumulative Growth in Sophisticated Ownership: Model (a) and Data (b).

(Proposition 3).

Expansion of Ownership In our dynamic exercise, we target the overall increase in sophisticated ownership. The expansion occurs in a very specific way across assets, both in the model and in the data. In the model, investors prefer to learn about assets with high volatility, and they initially start learning about the most volatile assets, which increases their holdings of those assets. Further increases in capacity induce them to expand learning to lower-volatility assets, per Lemma 3. In partial equilibrium, this process holds for both investor types. However, in general equilibrium, as sophisticated investors expand ownership, they take larger positions, which shrinks excess returns. Unsophisticated investors are more responsive to lower excess returns, and retrench.

As shown in the left panel of Figure 2, the model predicts that sophisticated investors exhibit the highest initial growth in ownership for the the highest-volatility assets, followed by growth in ownership of the medium-volatility assets, followed by growth for the lowest-volatility assets. This prediction is robustly borne out in the data, plotted the right panel of Figure 2.[19] We view this prediction as unique to our information-based mechanism, hence providing an important verification test of the model.

In Figure 3, we show *the change* in cross-sectional asset ownership between the two sub-

[19]To generate this graph in the model, we increase aggregate capacity from zero to the level that matches 51% sophisticated ownership, which is the last point in the data.

samples for sophisticated investors. We sort assets by the volatility of their returns. This cross-sectional change in ownership underlies the average ownership targets in the model of 23% in the initial period and 46% in the later period. Both the data and the model exhibit a hump-shaped profile of the increase; they are also very close quantitatively.

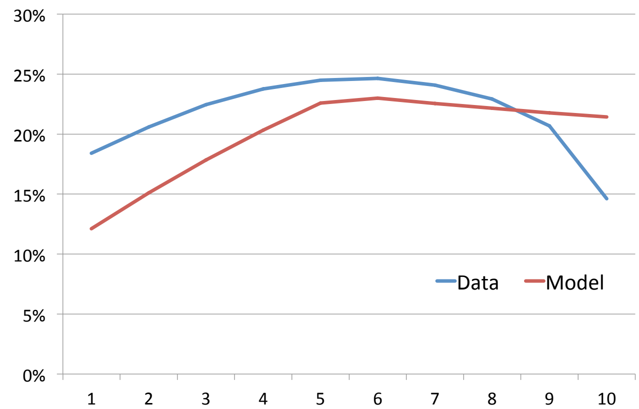


Figure 3: Absolute Change in Sophisticated Ownership by Asset Volatility Decile.

In conclusion, even though we parameterize the model to match the *aggregate* ownership levels of sophisticated investors in the pre- and post-2000 periods, the model also explains quantitatively *how* ownership changes *across* asset volatility classes, in terms of both the timing of growth levels and the absolute magnitudes of the changes.

Cross-sectional Turnover Our model implies cross-sectional variation in asset turnover. Intuitively, if an asset is more attractive and investors want to invest in it, then there are more investors with precise signals about this asset's returns, and these investors want to act on such better information by taking larger and more volatile positions. Since sophisticated investors receive more precise signals, and they have preference towards high-volatility assets, we should see a positive relationship between volatility and turnover. In Table 4, we report turnover in relation to return volatility in the model and in the data.

The first two rows compare data and the model prediction for 1989-2000 sub-sample. Both data and model show that turnover is increasing in volatility, and they are quantitatively close to each other. In the next two rows, we compare data for the 2001-2012 period to results generated from the dynamic exercise in the model in which we increase over-

Table 4: Turnover by Asset Volatility

Volatility quintile	1	2	3	4	5	Mean
1989-2000						
Data	5%	8.5%	10.5%	12.5%	11.5%	9.7%
Model	9.2%	9.3%	9.6%	10.1%	10.8%	9.7%
2001-2012						
Data	11%	14.6%	17%	18.4%	19.3%	16%
Model	12.3%	13.5%	14.3%	14.6%	14.8%	14%

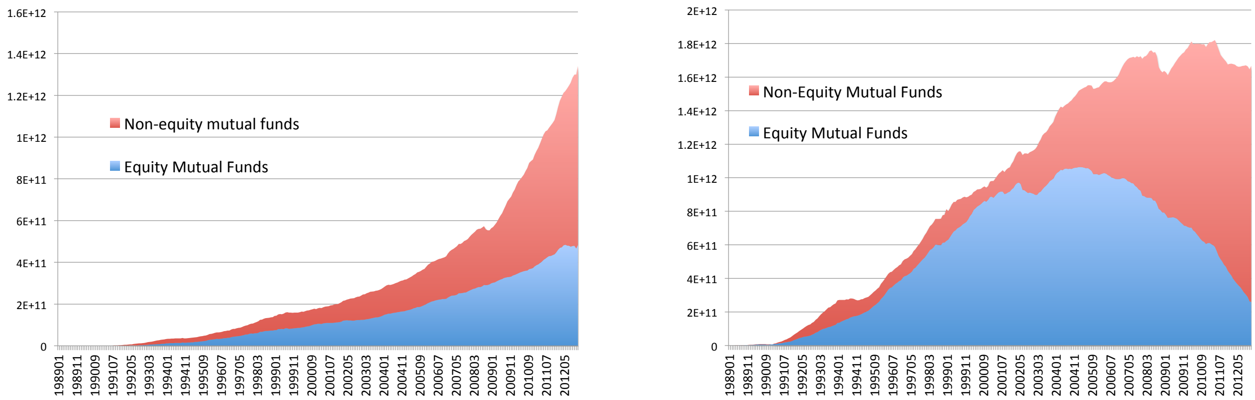
all capacity. The model implies an increase in average turnover and additionally matches the cross-sectional pattern of this increase. This effect is purely driven by our information friction, since the fundamental volatilities remain constant over time in this exercise.[20]

Retrenchment Across Other Asset Classes We provide auxiliary empirical support in favor of the model's ownership predictions by considering money flows into mutual funds. Equity funds are more risky than non-equity funds; hence, unsophisticated investors should be less likely to invest in the former, especially if aggregate information capacity grows.

We use mutual fund data from Morningstar, which classifies different funds into those serving institutional investors and individuals whose investment is at least $100,000 (institutional funds) and those serving individual investors with investment value less than $100,000 (retail funds). For the purpose of testing our predictions, we define sophisticated investors as those investing in institutional funds and unsophisticated investors as those investing in retail funds. We then calculate cumulative aggregate dollar flows into equity and non-equity funds, separately for each investor type. The data span the years 1989-2012.

As shown in Figure 4, the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the entire sample period. In contrast, the flows from unsophisticated investors display a markedly different pattern. The flows into equity funds grow until 2000 but subsequently decrease at a significant rate of more than 3 times by 2012.

[20]Our model also implies a positive turnover-ownership relationship, which we further confirm in the data. This result is consistent with the empirical findings in Chordia, Roll, and Subrahmanyam (2011).



(a) (b)

Figure 4: Cumulative Flows to Mutual Funds for Institutional (Sophisticated, panel a) and Retail (Unsophisticated, panel b) investors: Equity v. Non-Equity.

Moreover, this decrease coincides with a significant increase in cumulative flows to non-equity funds. Notably, the increase in equity fund flows by unsophisticated investors observed in the early sample period is consistent with the steady decrease in holdings of individual equity documented earlier. To the extent that direct equity holdings are more risky than diversified equity portfolios, such as mutual funds, this implies that unsophisticated investors have been systematically reallocating their wealth from riskier to safer asset classes.

Overall, these findings support the predictions of our model: Sophisticated investors have a large exposure to risky assets and subsequently add exposure to less risky assets, whereas unsophisticated investors leave riskier assets and increasingly move into safer assets as they perceive greater information disadvantage.

4 Quantifying Capital Income Inequality

In this section, we use our micro-level parametrization of the stochastic environment of the model to shed light on the main question of our paper: what drives the dynamics of capital income inequality? We show that our parameterized model, when mapped to household level data from the Survey of Consumer Finances (SCF), generates a path for capital income inequality that is quantitatively close to the data. The critical force in the model is *symmetric* aggregate technological growth combined with *initial* heterogeneity. We then show that the two key elements of the model, asset and investor heterogeneity, are essential to obtaining a good quantitative fit. Capital income inequality in the model is driven by *portfolio composition*–participation decisions in asset classes–rather than by pure return differential. We conclude with robustness checks.

4.1 Evidence from the Survey of Consumer Finances

We map investors in our model into households in the SCF. The SCF has been a standard testing ground for questions related to household finance and thus is a reliable source for our purpose. We restrict the sample to households who participate in capital markets, namely households with non-zero investment in stocks, bonds, or mutual funds, or with a brokerage account (34% of the SCF sample, on average). We use income flows from realized capital gains, dividend income, and interest income as our measure of capital income.[21]

A critical element for our analysis is the measurement of investor sophistication. Following the work of Arrow (1987), Calvet, Campbell, and Sodini (2009b), and Vissing-Jorgensen (2004), we use initial wealth levels as proxies for initial sophistication. We assume that wealthier individuals have access to better information production or processing technologies, i.e. they have greater information capacity. For each survey year, we consider two groups of participating households: those who are in the top decile of total wealth (sophisticated investors) and those who are in the bottom 50% of total wealth (unsophisticated investors).

Table 5 presents summary statistics for the 1989 and 2013 surveys. Capital income and financial wealth inequalities across the two sophistication groups are large, and exhibit substantial growth between 1989 and 2013 (Panel I). Notably, all the growth in financial wealth inequality is concentrated exclusively within our participant group. In Panel II, we show the financial wealth inequality for unsophisticated vs. non-participating group and for the bottom wealth decile of participants vs. non-participants. We find no significant increase in financial wealth inequality between non-participants and either of the participant groups.

[21] These correspond to variables 5706, 5708, 5710 and 5712 in the SCF.

Non-participants have more than twice the financial wealth of the bottom participants, and that number is stable in the data. In Figure 5, we present the time series for financial wealth inequality in the SCF for various household groups. The source of the inequality growth is concentrated within the participating group, and is most notable for our sophisticated vs. unsophisticated investor classes.

Sophisticated households earn more capital income per dollar of financial wealth (Panel III), a crude measure of their rates of return, which suggests that the composition of financial wealth of investor groups is different. Our measure of sophistication is also correlated with higher educational attainment and greater use of brokerage accounts (Panels IV and V).

The data also show a significant increase in access to brokerage accounts for unsophisticated relative to sophisticated investors. This fact, along with evidence that transaction costs on brokerage accounts have been trending down (French (2008)), suggests that the costs of accessing and transacting in financial markets are an unlikely explanation for the observed rise in capital income inequality. If anything, the improved access to financial markets should generate lower inequality, in the absence of informational heterogeneity.

Panel VI shows the fraction of financial wealth that each class of investors allocates to (low yield) liquid assets. Throughout the sample, sophisticated investors hold a much smaller fraction of their financial wealth in liquid assets. In turn, unsophisticated investors demonstrate a significant growth in the fraction of financial wealth held in liquid assets, from 33% in 1989 to 46% in 2013. This type of portfolio composition shift is consistent with our mechanism in Section 3.3. Panel VII reports the average age for each investor group. As expected, wealthy households are older on average. However, there are no time-series dynamics to the age difference that could explain the observed capital income dynamics.

4.2 Dynamics of Capital Income Inequality

We assess our model's quantitative predictions for the evolution of capital income inequality in response to aggregate growth in information technology. We use the ratio of financial wealth levels in the SCF in 1989 as a target for *initial* ratio of information capac-

Table 5: Investor Characteristics in the SCF

	1989	2013
I. Sophisticated/Unsophisticated Ratio		
Capital Income	61	129
Financial Wealth	38	66
II. Financial Wealth		
Unsophisticated/Non-participants	198%	219%
Bottom participants/Non-participants	42%	43%
III. Capital Income/Financial Wealth		
Sophisticated	10.7%	4.6%
Unsophisticated	9.2%	3.0%
IV. Highest Degree Earned		
Sophisticated	1.9	2.5
Unsophisticated	0.7	1.1
V. Has brokerage account		
Sophisticated	64%	82%
Unsophisticated	16%	35%
VI. Share of liquid assets in financial wealth		
Sophisticated	21%	19%
Unsophisticated	33%	46%
VII. Age (years)		
Sophisticated	58	60
Unsophisticated	49	51

Degree variable is coded as 0: $<$ 12 years; 1: jr. college or associate; 2: bachelor, nursing degree or other certificate; 3: master or MBA; 4: PhD, JD, MD, DDS/DMD, other doctorate. "Bottom participants" is the bottom decile of the wealth distribution in our participant group.

ities.[22] Subsequently, we assume that the growth rate of *aggregate* information capacity is determined by aggregate market return. Thus, all growth in income inequality is through portfolio composition decisions in response to aggregate progress in information technology.

The details of the parametrization of the new elements of the model are as follows. We set the initial ratio of investors' information capacity, K_1/K_2 in the model, to the 1989 ratio of average financial wealth in the top 10% and the bottom 50% of the total wealth distribution

[22] A guiding principle of our exercise is the existence of a capacity generating technology that is characterized by high fixed and low marginal costs, as explored in Arrow (1987).

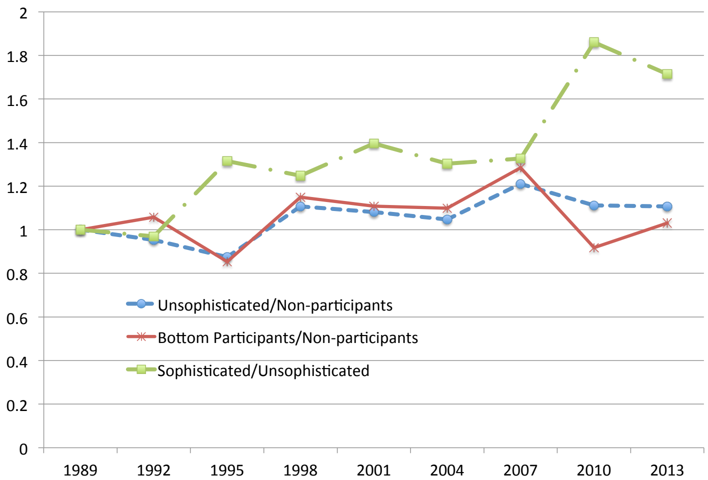


Figure 5: Financial Wealth Inequality: Extensive Margin of Participation in the SCF.

of our households. In the data, this ratio is equal to 38. We then pick the initial aggregate capacity level to match the excess return on the market portfolio, equal to 11.9% in the data,[23] and assume that the growth of each investor type's capacity is the same and equal to the market return. We simulate the model for 25 years forward, which is the time span of our data set. The outcome of the experiment is the endogenous capital income inequality growth implied by our mechanism.

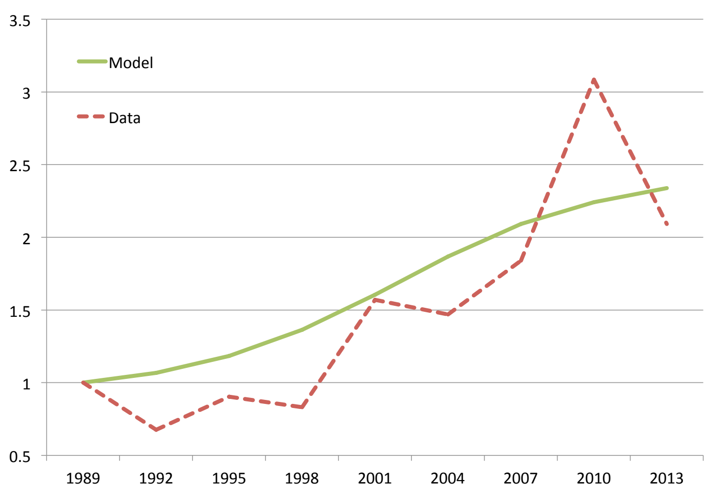


Figure 6: Cumulative Growth in Capital Income Dispersion

The results of this exercise are presented in Figure 6. The model comes very close to

[23]We also set initial wealth to match the average initial return on wealth of 10%, consistent with the SCF in 1989. The parametrization procedure gives capacity levels equal to $K_1 = 0.3169$ and $K_2 = 0.0083$.

matching the overall growth in inequality in the data, with a 134% growth in the model vs. 109% growth in the data.

The Role of Heterogeneity Below, we explore a quantitative importance of heterogeneity in investor capacity and asset heterogeneity for the model's quantitative predictions. In Figure 7, we present results from two alternative specifications of the benchmark model.

In the first specification, labeled *Asymmetric Growth*, we ask how important are initial capacity differences versus different capacity evolution for capital income inequality. In particular, we consider growth in capacity driven by *individual* rather than market returns on equity. This generates asymmetric growth in capacities across investor groups. As Figure 7 demonstrates, asymmetric capacity growth driven by individual returns increases the growth in inequality over time. Nevertheless, the quantitative impact is small relative to the benchmark inequality growth. This indicates that the *initial* capacity heterogeneity combined with subsequent aggregate growth are the key forces driving the evolution of inequality in the model.

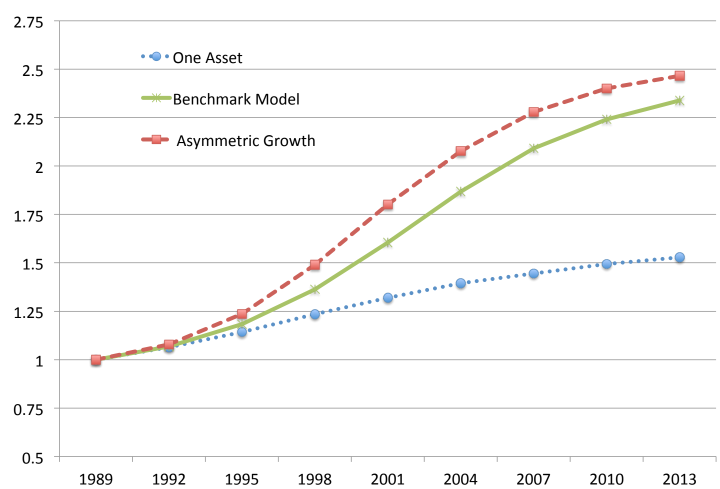


Figure 7: Cumulative Growth in Capital Income Dispersion: The Role of Asset and Capacity Growth Heterogeneity.

In the second specification, labeled *One Asset*, we quantify the role of asset heterogeneity in driving capital income inequality. Specifically, we consider an analogously parameterized model with only one risky asset.[24] The one-asset economy generates growth in capital income inequality that is approximately 40% of the growth generated by the benchmark model. Hence, asset heterogeneity plays a crucial role in driving capital income inequality in the model. It generates higher payoffs from learning and larger effects on the retrenchment of unsophisticated investors from risky asset markets.

4.3 Robustness

The Role of Capital Income in Financial Wealth To assess the importance of capital income as a driving force of financial wealth inequality, relative to other mechanisms, such as savings rates from non-capital income sources, we generate the counterfactual financial wealth obtained from reinvesting capital income only. Specifically, for each wealth decile in the 1989 SCF, we take 1989 financial wealth as a starting point, and derive a hypothetical wealth level in subsequent SCF surveys by accumulating capital income.[25]

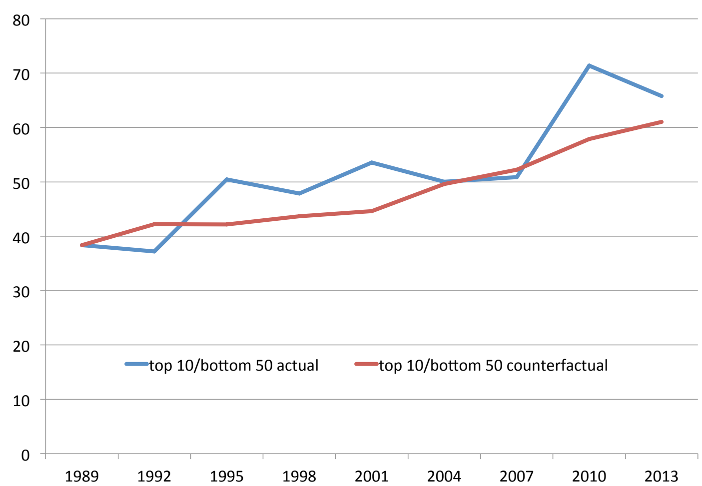


Figure 8: Financial Wealth: Actual and counterfactual due only to capital income.

[24] In terms of the parametrization, the model with one asset takes away two targets from the benchmark model: heterogeneity in asset volatility and fraction of actively traded assets. We keep the value of risk aversion coefficient the same as in the benchmark model and change only three parameters: overall capacity ϕ, volatility of the noise trader demand σ_x, and the volatility of asset payoff σ to match: the average market return (11.9%), asset turnover (9.7%), and sophisticated ownership (23%). To make the results comparable, in simulating the model, we feed in aggregate capacity growth equal to the one in the benchmark model.

[25] For example, the counterfactual financial wealth level in 1995 is equal to financial wealth in 1989 plus 3 times capital income in the 1989 and 1992 surveys.

Figure 8 shows the time series for actual and counterfactual financial wealth inequality between sophisticated and unsophisticated investors. The two series are remarkably close, which suggests an important role for capital income in the evolution of financial wealth.[26] One interpretation of Figure 8 is that looking at past capital income realizations may be sufficient to explain the evolution of financial wealth, without resorting to mechanisms that incorporate savings rates from other income sources. Still, we treat this evidence as suggestive only, since this construction imposes a panel interpretation on a repeated cross-section.

Passive Investment Policies We study whether capital income differences are an outcome of differences in *market returns* over time combined with *passive*, buy-and-hold household strategies. The hypothesis is that some households (the wealthy) hold a larger share of their wealth in stock relative to the median household, which gives higher returns by the mere fact that stocks outperform bonds. Figure 9 plots, for each year, the past 15-year cumulative return on holding the aggregate index of the U.S. stock market.[27] This gives the cumulative return of such passive strategy of a household, relative to a household which exclusively holds bonds (with a gross return of 1). The cumulative return on the passive strategy actually exhibits a declining trend, which implies that if investors used the passive strategy and the only difference was how much money they hold in the stock market versus bonds, then we should observe a declining trend in capital income inequality, as the gross return on the market converges to the gross return on bonds. This exercise highlights the importance of active decisions of when to enter and exit stock market in generating returns.

Endogenous Capacity Choice In the benchmark model, we assume an exogenous relation between initial capacity and an investor's wealth. In the Appendix, we show how such relation could arise endogenously. Intuitively, if investors endogenously choose different portfolio sizes, then their net benefit of investing in information will increase with portfolio size. We apply this idea in a model in which investors have identical CRRA preferences and

[26]By construction, the two wealth levels are identical in 1989, so the figure also implies that the counterfactual levels of financial wealth of each of the group are very close to the data.

[27]The patterns we document are essentially the same for other choices of the horizon: 5, 10, or 20 years.

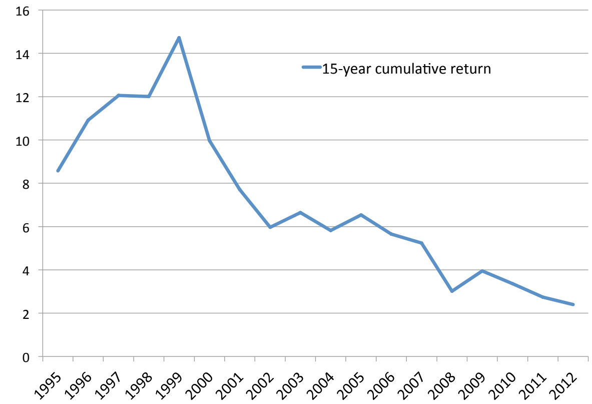


Figure 9: Cumulative market return on a 15-year passive investment in the U.S. stock market.

make endogenous capacity choice decisions. In the context of the information choice model, CRRA utility specification is not tractable; hence, we map a common relative risk aversion together with wealth differences locally into different absolute risk aversion coefficients. In a numerical example, we show how initial wealth differences observed in the 1989 SCF map into endogenous capacity differences, for different values of the cost of capacity and different relative risk aversion coefficients. We show that for a wide range of the risk aversion specifications and for capacity cost away from zero, the implied differences in capacity are equal or actually larger than the ones specified in the benchmark model. Hence, we view our parametrization as cautious in that it implies modest initial differences.

Constant Relative Risk Aversion Utility In the Appendix, we analyze the model with CRRA utility. Since a closed-form solution to the full model is not feasible, we focus on a local approximation of the utility function. We show that the model solution under no capacity differences predicts the same portfolio shares for risky assets, *independent of wealth*. Intuitively, if agents have common information, then wealth differences affect the composition of their allocation between the risk-free asset and the risky portfolio, but not the composition of the risky portfolio, which is determined optimally by the (common) belief structure. As a result, differences in capacity are a necessary component of the model to generate any risky return differences across agents.

5 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with the seminal work by Kuznets (1953). We approach this question from the perspective of capital income that is known to be highly unequally distributed across individuals. We propose a theoretical information-based framework that links capital income derived from financial markets to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total sophistication in the market, and could be the result of aggregate technological progress. Additional predictions on asset ownership, market returns, and turnover help us pin down the economic mechanism and rule out alternative explanations. The quantitative predictions of the model match qualitatively and quantitatively the observed data.

One could argue that the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market. However, our work suggests that one should assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. Our work also sheds light on the overall benefits and redistribution aspects of progress in financial markets in terms of creating new financial instruments. Depending on where the new assets land on the volatility (or more generally, opaqueness) spectrum, the benefits will accrue to the relatively less (low-volatility assets) or more (high-volatility assets) sophisticated investors.

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Appendix: Proofs

Model

Portfolio Choice. In the second stage, each investor chooses portfolio holdings q_{ji} to solve

$\max_{\{q_{ji}\}_{i=1}^{n}} U_j = E_j(W_j) - \frac{\rho}{2} V_j(W_j) \quad s.t. \;\; W_j = r\left(W_{0j} - \sum_{i=1}^{n} q_{ji} p_i\right) + \sum_{i=1}^{n} q_{ji} z_i,$

where E_j and V_j denote the mean and variance conditional on investor j's information set:

$E_j(W_j) = E_j\left[rW_{0j} + \sum_{i=1}^{n} q_{ji}(z_i - rp_i)\right] = rW_{0j} + \sum_{i=1}^{n} q_{ji}\left[E_j(z_i) - rp_i\right],$

$V_j(W_j) = V_j\left[rW_{0j} + \sum_{i=1}^{n} q_{ji}(z_i - rp_i)\right] = \sum_{i=1}^{n} q_{ji}^2 V_j(z_i).$

Let $\widehat{\mu}_{ji} \equiv E_j[z_i]$ and $\widehat{\sigma}_{ji}^2 \equiv V_j\,[z_i]$. The investor's portfolio problem is to maximize

$U_j = rW_{0j} + \sum_{i=1}^{n} q_{ji}(\widehat{\mu}_{ji} - rp_i) - \frac{\rho}{2} \sum_{i=1}^{n} q_{ji}^2 \widehat{\sigma}_{ji}^2.$

The first order conditions with respect to q_{ji} yield $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \widehat{\sigma}_{ji}^2}$. Since W_{0j} does not affect the optimization, we normalize it to zero. The indirect utility function becomes

$U_j = \frac{1}{2\rho} \sum_{i=1}^{n} \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}.$ □

Posterior Beliefs. The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that $Cov(s_{ji}, z_i) = \sigma_{sji}^2$ and

$\widehat{\mu}_{ji} = \overline{z} + \frac{Cov(s_{ji}, z_i)}{\sigma_{sji}^2}(s_{ji} - \overline{s}_{ji}) = s_{ji},$

$\widehat{\sigma}_{ji}^2 = \sigma_i^2 \left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \sigma_{\delta ji}^2.$ □

Information Constraint. Let $H(z)$ denote the entropy of z, and let $H(z|s_j)$ denote the conditional entropy of z given the vector of signals s_j. Then

$I(z; s_j) \equiv H(z) - H(z|s_j) \overset{(1)}{=} \sum_{i=1}^{n} H(z_i) - H(z|s_j) \overset{(2)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H\left(z_i | z^{i-1}, s_j\right)$

$\overset{(1)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H(z_i|s_j) \overset{(3)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H(z_i|s_{ji}) = \sum_{i=1}^{n} I(z_i; s_{ji})$

where (1) follows from the independence of the payoffs z_i; (2) follows from the chain rule for entropy, where $z^{i-1} = \{z_1, ..., z_{i-1}\}$; (3) follows from the independence of the signals s_{ji}.

For each asset i, the entropy of $z_i \sim \mathcal{N}\left(\overline{z}, \sigma_i^2\right)$ is $H(z_i) = \frac{1}{2}\ln\left(2\pi e \sigma_i^2\right).$

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$I(z_i; s_{ji}) = H(z_i) + H(s_{ji}) - H(z_i, s_{ji}) = \frac{1}{2}\log\left(\frac{\sigma_i^2 \sigma_{sji}^2}{|\Sigma_{z_i s_{ji}}|}\right) = \frac{1}{2}\log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right),$

where $\left|\Sigma_{z_i s_{ji}}\right| = \sigma_{sji}^2 \sigma_{\delta ji}^2$ is the determinant of the variance-covariance matrix of z_i and s_{ji}.

Hence $I(z_i; s_{ji}) = 0$ if $\sigma_{\delta ji}^2 = \sigma_i^2$. Note, for an *additive noise* signal structure, $\widetilde{s}_{ji} = z_i + \widetilde{\delta}_{ji}$, $I(z_i; \widetilde{s}_{ji}) = \frac{1}{2}\log\left(\frac{\sigma_i^2 + \widetilde{\sigma}_{\delta ji}^2}{\widetilde{\sigma}_{\delta ji}^2}\right)$. Hence $I(z_i; \widetilde{s}_{ji}) \to 0$ as $\widetilde{\sigma}_{\delta ji}^2 \to \infty$.

Across assets, $I(z; s_j) = \sum_{i=1}^n I(z_i; s_{ji}) = \frac{1}{2}\sum_{i=1}^n \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) = \frac{1}{2}\log\left(\prod_{i=1}^n \frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) \leq K_j$.

Finally, since $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$, the information constraint becomes $\prod_{i=1}^n \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}$. □

Information Objective. Expected utility is given by

$$E_{0j}[U_j] = \frac{1}{2\rho} E_{0j}\left[\sum_{i=1}^n \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}\right] = \frac{1}{2\rho}\sum_{i=1}^n \frac{E_{0j}\left[(\widehat{\mu}_{ji} - rp_i)^2\right]}{\widehat{\sigma}_{ji}^2} = \frac{1}{2\rho}\sum_{i=1}^n \left(\frac{\widehat{R}_{ji}^2 + \widehat{V}_{ji}}{\widehat{\sigma}_{ji}^2}\right),$$

where $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $\widehat{\mu}_{ji} - rp_i$. Conjecture (and later verify) that prices are normally distributed, $p_i \sim \mathcal{N}\left(\overline{p}_i, \sigma_{pi}^2\right)$.

$$\widehat{R}_{ji} \equiv E_{0j}(\widehat{\mu}_{ji} - rp_i) = \overline{z} - r\overline{p}_i,$$

$$\widehat{V}_{ji} \equiv V_{0j}(\widehat{\mu}_{ji} - rp_i) = Var(\widehat{\mu}_{ji}) + r^2\sigma_{pi}^2 - 2rCov(\widehat{\mu}_{ji}, p_i).$$

The signal structure implies that $Var(\widehat{\mu}_{ji}) = \sigma_{sji}^2$.

Following Admati (1985), we conjecture (and later verify) that prices are $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$. We compute $Cov(\widehat{\mu}_{ji}, p_i)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs:

$$Cov(\widehat{\mu}_{ji}, p_i) = \frac{Cov(\widehat{\mu}_{ji}, z_i)Cov(z_i, p_i)}{\sigma_i^2}.$$

Since $Cov(z_i, p_i) = b_i\sigma_i^2$ and $Cov(\widehat{\mu}_{ji}, z_i) = \sigma_{sji}^2$, then $Cov(\widehat{\mu}_{ji}, p_i) = b_i\sigma_{sji}^2$. Then

$$\widehat{V}_{ji} = \sigma_{sji}^2 + r^2\sigma_{pi}^2 - 2rb_i\sigma_{sji}^2 = (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2.$$

Hence the distribution of expected excess returns is normal with mean and variance:

$$\widehat{R}_{ji} = \overline{z} - ra_i \quad \text{and} \quad \widehat{V}_{ji} = (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2.$$

Expected utility becomes

$$E_{0j}[U_j] = \frac{1}{2\rho}\sum_{i=1}^n \left[\frac{(\overline{z} - ra_i)^2 + (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2}{\widehat{\sigma}_{ji}^2}\right] = \frac{1}{2\rho}\sum_{i=1}^n G_i\frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} - \frac{1}{2\rho}\sum_{i=1}^n (1 - 2rb_i),$$

where $G_i \equiv (1 - rb_i)^2 + \frac{r^2c_i^2\sigma_x^2}{\sigma_i^2} + \frac{(\overline{z} - ra_i)^2}{\sigma_i^2}$, and where the second summation is independent of the investor's choices. □

Proof of Lemma 1. The linear objective function and the convex constraint imply that each investor allocates all capacity to learning about a single asset. For all other assets, the posterior variance is equal to the prior variance. Let l_j index the asset about which investor j learns. The information constraint becomes $\prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \frac{\sigma_{l_j}^2}{\widehat{\sigma}_{jl_j}^2} = e^{2K_j}$, and hence the variance of the investor's beliefs is given by

$$\widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases}$$

The investor's problem becomes picking the asset l_j to maximize $\sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \left(e^{2K_j} - 1\right) G_{l_j} + \sum_{i=1}^{n} G_i$. Since $e^{2K_j} > 1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j \in \arg\max_i G_i$. The distribution of posterior beliefs follows. □

Conditional Distribution of Signals. Conditional on the realized payoff, the signal is a normally distributed random variable, with mean and variance given by

$$E\left(s_{ji}|z_i\right) = \overline{s}_{ji} + \frac{Cov(s_{ji}, z_i)}{\sigma_i^2}\left(z_i - \overline{z}\right) = \begin{cases} \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j, \end{cases}$$

$$V\left(s_{ji}|z_i\right) = \sigma_{sji}^2\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2\sigma_i^2}\right) = \begin{cases} \left(1 - e^{-2K_j}\right)e^{-2K_j}\sigma_i^2 & \text{if } i = l_j \\ 0 & \text{if } i \neq l_j. \end{cases}$$ □

Proof of Lemma 2. The market clearing condition for each asset in state (z_i, x_i) is

$$\int_{M_{1i}}\left(\frac{s_{ji} - rp_i}{e^{-2K_1}\rho\sigma_i^2}\right)dj + \int_{M_{2i}}\left(\frac{s_{ji} - rp_i}{e^{-2K_2}\rho\sigma_i^2}\right)dj + \left(1 - m_{1i} - m_{2i}\right)\left(\frac{\overline{z} - rp_i}{\rho\sigma_i^2}\right) = x_i,$$

where M_{1i} denotes the set of measure $m_{1i} \in [0, \lambda]$ of sophisticated investors who choose to learn about asset i, and M_{2i} denotes the set of measure $m_{2i} \in [0, 1-\lambda]$, of unsophisticated investors who choose to learn about asset i.

Using the conditional distribution of the signals, $\int_{M_{1i}} s_{ji}dj = m_{1i}\left[\overline{z} + \left(1 - e^{-2K_1}\right)\varepsilon_i\right]$ for the type-1 investors, and analogously for the type-2 investors. Then, the market clearing condition can be written as $\alpha_1\overline{z} + \alpha_2\varepsilon_i - x_i = \alpha_1 rp_i$, where

$$\alpha_1 \equiv \frac{1 + m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho\sigma_i^2} \text{ and } \alpha_2 \equiv \frac{m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho\sigma_i^2}.$$

We obtain identification of the coefficients in $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$ as

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\overline{x}}{\alpha_1}\right],\ b_i = \frac{\alpha_2}{r\alpha_1}, \text{ and } c_i = \frac{1}{r\alpha_1}.$$

Let $\Phi_i \equiv m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)$ be a measure of the information capacity allocated to learning about asset i in equilibrium. Further substitution yields

$$a_i = \frac{1}{r}\left(\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{1 + \Phi_i}\right), \quad b_i = \frac{1}{r}\left(\frac{\Phi_i}{1 + \Phi_i}\right), \quad c_i = \frac{1}{r}\left(\frac{\rho\sigma_i^2}{1 + \Phi_i}\right).$$ □

Proof of Lemma 3. Substituting a_i, b_i, and c_i, equilibrium gains become $G_i = \frac{1+\rho^2\sigma_i^2\left(\sigma_x^2+\overline{x}^2\right)}{\left(1+\Phi_i\right)^2}$. Defining $\xi_i \equiv \sigma_i^2\left(\sigma_x^2+\overline{x}^2\right)$, gives $G_i = \frac{1+\rho^2\xi_i}{\left(1+\Phi_i\right)^2}$.

By Lemma 1, each investor learns about a single asset among the assets with the highest gain. WLOG, assets in the economy are ordered such that $\sigma_i > \sigma_{i+1}$, for all $i \in \{1,...,n-1\}$. First suppose that all investors learn about the same asset. Since G_i is increasing in σ_i, this asset is asset 1. All investors learn about a single asset as long as $\phi \leq \phi_1 \equiv \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_2}} - 1$. At this threshold, some investors switch and learn about the second asset.

For $\phi > \phi_1$, equilibrium gains must be equated among all assets with positive learning mass. Otherwise, investors have an incentive to switch to learning about the asset with the higher gain. Moreover, the gains of all assets with zero learning mass must be strictly lower. Otherwise, an investor would once again have the incentive to deviate and learn about one of these assets.

We now derive expressions for the mass of investors learning about each asset. We assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population: $m_{1i} = \lambda m_i$ and $m_{2i} = (1-\lambda)\, m_i$, where m_i is the total mass of investors learning about asset i. Hence $\Phi_i = \phi m_i$. Note that this implies that the masses m_i are also strictly decreasing in i across the assets that are learned about. We can write the necessary and sufficient conditions for determining $\{m_i\}_{i=1}^n$ as $\sum_{i=1}^k m_i = 1$; $\frac{1+\phi m_i}{1+\phi m_1} = c_{i1}$, for any $i \in \{2,...,k\}$, where $c_{i1} \equiv \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_1}} \leq 1$, with equality *iff* $i = 1$; and $m_i = 0$ for any $i \in \{k+1,...,n\}$. Recursively, $m_i = c_{i1}m_1 - \frac{1}{\phi}\left(1-c_{i1}\right), \quad \forall i \in \{2,...,k\}$. Using $\sum_{i=1}^k m_i = 1$, and defining $C_k \equiv \sum_{i=1}^k c_{i1}$, we obtain the solution for m_1 given by $m_1 = \frac{1}{C_k} + \frac{1}{\phi}\left(\frac{k}{C_k} - 1\right)$, where we have used the fact that $c_{11} = 1$. Using this expression, we obtain the solution for all m_i, $i \in \{1,...,k\}$, $m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi}\left(\frac{kc_{i1}}{C_k} - 1\right)$. □

Proof of Lemma 4. **(i)** First, consider an increase in ϕ to some $\phi' \leq \phi_k$, such that no new assets are learned about in equilibrium (k and C_k are unchanged). For $i \in \{1,...,k\}$,

$$\frac{dm_i}{d\phi} = -\frac{1}{\phi^2}\left(\frac{kc_{i1}}{C_k} - 1\right).$$

Hence m_i is strictly decreasing in ϕ if $c_{i1} > \frac{C_k}{k}$ (namely, if the asset is above average in terms of adjusted volatility), and m_i is increasing in ϕ otherwise. Since c_{i1} is decreasing in i, the condition $c_{\overline{\imath}1} = C_k/k$ defines the cutoff asset $\overline{\imath}$. Moreover, note that for $i \in \{1,...,\overline{\imath}\}$, the absolute value of $\frac{dm_i}{d\phi}$ is decreasing in i, such that the masses of the more volatile assets fall by more than those of the less volatile assets. Likewise, for $i \in \{\overline{\imath}+1,...,k\}$, the value of $\frac{dm_i}{d\phi}$ is increasing in i, such that the masses of the less volatile assets increase by more than those of the more volatile assets. This results in a flattening of the distribution of investors across assets.

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about (with $k' \leq n$). Let the new equilibrium masses be denoted by m_i' for $i \in \{1,...,k'\}$. Hence, $\Sigma_{i=1}^k m_i' < 1$. Using the recursive expression for m_i in terms of m_1, for $i \in \{2,...,k\}$

$$m_i - m_i' = c_{i1}\left(m_1 - m_1'\right) - \left(1-c_{i1}\right)\left(\frac{1}{\phi} - \frac{1}{\phi'}\right).$$

Suppose that $m_1 \leq m_1'$. Then $m_i - m_i' < 0$, which implies that $\Sigma_{i=1}^k m_i - \Sigma_{i=1}^k m_i' = 1 - \Sigma_{i=1}^k m_i' < 0$, which is a contradiction. Hence $m_1 > m_1'$. Moreover, since c_{i1} is decreasing in i, the condition $m_{\bar{\imath}} = m_{\bar{\imath}}'$ defines the threshold value for c_{i1} that defines the cutoff asset $\bar{\imath}$.

(ii) First, consider an increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about (k and C_k are unchanged). For $i \in \{1, ..., k\}$,

$\frac{d(\phi m_i)}{d\phi} = \frac{c_{i1}}{C_k} > 0.$

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about in equilibrium (with $k' \leq n$). First, for the new assets that are actively traded, $i \in \{k+1, ..., k'\}$, $m_i' > m_i = 0$, hence, $\phi' m_i' > \phi m_i$. Second, consider an asset $i \in \{1, ..., k\}$ and an asset $h \in \{k+1, ..., k'\}$. Let the new equilibrium gains be denoted by G_i' and G_h'. Then $G_i > G_h$, which implies that $1 + \phi m_i < c_{ih}$, and $G_i' = G_h'$, which implies that $1 + \phi' m_i' = (1 + \phi' m_h') c_{ih} > (1 + \phi' m_h')(1 + \phi m_i) \Leftrightarrow \phi' m_i' > \phi m_i + \phi' m_h' (1 + \phi m_i) > \phi m_i$.

(iii) Let $K_1 = K$ and $K_2 = \delta K$, for some $\delta \in (0,1)$, and consider an increase in K such that the first $k' \geq k$ assets are learned about. For $i \in \{k+1, ..., k'\}$, first, $m_i = 0$ and $m_i' > 0$; second, $d(e^{2K} - 1)/dK = 2e^{2K} > 2\delta e^{2K\delta} = d(e^{2K\delta} - 1)/dK > 0$. The result follows.

For $i \in \{1, ..., k\}$, let $m_{i\phi} \equiv \frac{dm_i}{d\phi}$. The derivatives of interest are

$D_1 \equiv \frac{d[m_i(e^{2K}-1)]}{dK} = m_{i\phi}\left(e^{2K} - 1\right)\frac{d\phi}{dK} + 2e^{2K} m_i$

$D_2 \equiv \frac{d[m_i(e^{2K\delta}-1)]}{dK} = m_{i\phi}\left(e^{2K\delta} - 1\right)\frac{d\phi}{dK} + 2\delta e^{2K\delta} m_i$

where $\frac{d\phi}{dK} = 2\lambda e^{2K} + 2\delta\left(1-\lambda\right)e^{2K\delta} > 0$.

First, for assets $i \in \{\bar{\imath}, ..., k\}$, for which $m_{i\phi} \geq 0$, $D_1 > D_2 > 0$, since $e^{2K} > e^{2K\delta} > \delta e^{2K\delta}$.

Next, for assets $i \in \{1, ..., \bar{\imath} - 1\}$, for which $m_{i\phi} < 0$, factoring out $2e^{2K}$ yields

$$\begin{aligned}
D_1 &= 2e^{2K}\left\{m_i + m_{i\phi}\left(e^{2K}-1\right)\left[\lambda + \left(1-\lambda\right)\delta e^{2K(\delta-1)}\right]\right\} \\
&= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + \left(1-\lambda\right)\delta\left(e^{2K\delta} - e^{2K(\delta-1)}\right)\right]\right\} \\
&> 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + \left(1-\lambda\right)\left(e^{2K\delta}-1\right)\right]\right\} \\
&= 2e^{2K}\left\{m_i + m_{i\phi}\phi\right\} = 2e^{2K}\left[\frac{d(\phi m_i)}{d\phi}\right] > 0,
\end{aligned}$$

where the first inequality follows from $m_{i\phi} < 0$, $\delta < 1$, $e^{2K} > 1$, and $e^{2K(\delta-1)} < 1$; and the last inequality follows from part (ii) above.

Next, note that $\lambda D_1 + (1-\lambda)D_2 = \left[\frac{d(\phi m_i)}{d\phi}\right]\frac{d\phi}{dK} = 2\left[\frac{d(\phi m_i)}{d\phi}\right]\left[\lambda e^{2K} + \delta\left(1-\lambda\right)e^{2K\delta}\right]$. We have just shown that $D_1 > 2e^{2K}\left[\frac{d(\phi m_i)}{d\phi}\right]$, so for the equality to hold, it must be that $D_2 < 2\delta e^{2K\delta}\left[\frac{d(\phi m_i)}{d\phi}\right]$. Hence, $D_1 > 0$ and $D_1 > D_2$. It remains to be determined if $D_2 > 0$ as well. We can obtain a sufficient condition for $D_2 > 0$ as follows: For $m_{i\phi} < 0$,

$D_2 = 2\delta e^{2K\delta} m_i + 2m_{i\phi}\left(e^{2K\delta} - 1\right)\left[\lambda e^{2K} + \left(1-\lambda\right)\delta e^{2K\delta}\right]$

$$
\begin{aligned}
&= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-e^{2K(1-\delta)}\right)+(1-\lambda)\,\delta\left(e^{2K\delta}-1\right)\right]\right\}. \\
&> 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-e^{2K(1-\delta)}\right)+(1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&> 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right)+(1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\phi\right\} = 2e^{2K\delta}\left\{\left[\frac{d(\phi m_i)}{d\phi}\right]-(1-\delta)\,m_i\right\},
\end{aligned}
$$

where the first inequality follows from $m_{i\phi} < 0$ and $\delta < 1$, and the second inequality follows from $m_{i\phi} < 0$ and $e^{2K(1-\delta)} > 1$. Hence if δ is not too small (i.e. capacity dispersion is not too large), then $D_2 > 0$ for $i \in \{1, ..., \bar{\imath} - 1\}$ as well.

Summarizing, in response to symmetric capacity growth, for assets $i \in \{\bar{\imath}, ..., k'\}$, both $m_i\left(e^{2K_1}-1\right)$ and $m_i\left(e^{2K_2}-1\right)$ growth, but $m_i\left(e^{2K_1}-1\right)$ grows by more. For assets $i \in \{1, ..., \bar{\imath}-1\}$, for which the mass of investors falls in response to the capacity growth, $m_i\left(e^{2K_1}-1\right)$ grows and $m_i\left(e^{2K_2}-1\right)$ grows by less, or even falls, if capacity dispersion is large enough. □

Analytic Results

Proof of Proposition 1. Results follow from equations (14-16). □

Proof of Proposition 2. **(i)** Follows from the definition of capital income per capita and equation (15). **(ii)** Since for all $i \in \{1, ..., k\}$, the gains G_i are equated in equilibrium, then $E\left[\pi_{1i} - \pi_{2i}\right]$ is increasing in m_i, which in turn is increasing in σ_i^2. □

Derivation of volume per capita. Consider an investor with asset holdings q in period t, and let f denote the PDF and F the CDF of the cross-sectional distribution of holdings in this investor's group, with mean $\overline{q}$ and standard deviation σ. Since shocks are i.i.d., if investors don't change groups over time, the distribution of q in each investor group is stationary. The investor's expected volume of trade from t to $t+1$ is

$$
\begin{aligned}
v \equiv \int_{-\infty}^{\infty} |q'-q|\, f(q')\, dq' &= \int_{-\infty}^{q} (q-q')\, f(q')\, dq' + \int_{q}^{\infty} (q'-q)\, f(q')\, dq' \\
&= \int_{-\infty}^{q} q f(q')\, dq' - \int_{-\infty}^{q} q' f(q')\, dq' + \int_{q}^{\infty} q' f(q')\, dq' - \int_{q}^{\infty} q f(q')\, dq'.
\end{aligned}
$$

Adding and subtracting terms gives $v = q\left[2F(q)-1\right] + \overline{q} - 2F(q)\, E\left[q'|q' < q\right]$.

Using $E\left[q'|q' < q\right] = \overline{q} - \sigma^2\left[\frac{f(q)}{F(q)}\right]$, $v = q\left[2F(q)-1\right] + \overline{q} - 2F(q)\,\overline{q} + 2\sigma^2 f(q)$.

Averaging across all q in the group,

$$V = 2\int_{-\infty}^{\infty} qF(q)\, f(q)\, dq - 2\overline{q}\int_{-\infty}^{\infty} F(q)\, f(q)\, dq + 2\sigma^2\int_{-\infty}^{\infty} f(q)^2\, dq.$$

Using the formulas for integrals of normal distributions,

$\int_{-\infty}^{\infty} F(q)\, f(q)\, dq = 1/2$, $\int_{-\infty}^{\infty} qF(q)\, f(q)\, dq = \overline{q}/2 + \sigma/\left(2\sqrt{\pi}\right)$, and $\int_{-\infty}^{\infty} f(q)^2\, dq = 1/2\sigma\sqrt{\pi}$.

Then, the per capita expected volume of trade is

$$V = \overline{q} + \frac{\sigma}{\sqrt{\pi}} - \overline{q} + \frac{\sigma}{\sqrt{\pi}} = \frac{2\sigma}{\sqrt{\pi}}.$$ □

Derivation of asset turnover. Consider the group of sophisticated investors actively trading asset i. A particular investor j in this group holds $q_{ji} = e^{2K_1}\left(s_{ji} - rp_i\right)/\rho\sigma_i^2$. Conditional on the state (z_i, x_i), the cross-sectional variance of holdings for this group is

$Var\left(q_{ji}\right) = \frac{e^{4K_1}}{\rho^2\sigma_i^4}Var\left(s_{ji} - rp_i\right) = \frac{e^{2K_1}-1}{\rho^2\sigma_i^2}$.

Hence, the per capita expected volume for active sophisticated investors is $V_{1i} = \frac{2}{\sqrt{\pi}}\left(\frac{\sqrt{e^{2K_1}-1}}{\rho\sigma_i}\right)$ and for active unsophisticated investors is $V_{2i} = \frac{2}{\sqrt{\pi}}\left(\frac{\sqrt{e^{2K_2}-1}}{\rho\sigma_i}\right)$.

Next, consider the group of sophisticated investors passively trading asset i. A particular investor j in this group holds $q_{ji} = \left(\overline{z} - rp_i\right)/\rho\sigma_i^2$. Conditional on the realized state (z_i, x_i), the cross-sectional variance of holdings for this group is 0. Hence, the per capita expected volume for passive sophisticated investors is $V_{3i} = 0$. Analogously, the per capita expected volume for passive unsophisticated investors is $V_{4i} = 0$. This gives, the expected volume for asset i,

$V_i = \lambda m_i\frac{2}{\sqrt{\pi}}\left(\frac{\sqrt{e^{2K_1}-1}}{\rho\sigma_i}\right) + (1-\lambda)\,m_i\frac{2}{\sqrt{\pi}}\left(\frac{\sqrt{e^{2K_2}-1}}{\rho\sigma_i}\right) = \frac{2m_i}{\sqrt{\pi}}\left[\frac{\lambda\sqrt{e^{2K_1}-1}+(1-\lambda)\sqrt{e^{2K_2}-1}}{\rho\sigma_i}\right]$,

and average turnover

$T_i \equiv \frac{V_i}{\bar{x}} = \frac{2m_i}{\bar{x}\sqrt{\pi}}\left[\frac{\lambda\sqrt{e^{2K_1}-1}+(1-\lambda)\sqrt{e^{2K_2}-1}}{\rho\sigma_i}\right]$. □

Proof of Proposition 3. Results follow from the expression for asset turnover derived above. □

Proof of Proposition 4. **(i)** The increase in dispersion keeps ϕ unchanged. Therefore, using equation (11), the masses m_i are unchanged. With both ϕ and m_i unchanged, prices are unchanged. **(ii)** The result follows from equation (15): masses and prices do not change, and dispersion, $\left(e^{2K_1} - e^{2K_2}\right)$ increases. **(iii)** Relative capital income is

$$\frac{\pi_{1i}}{\pi_{2i}} = \frac{\left(\overline{z}_i - rp_i\right)\left(z_i - rp_i\right) + \left(e^{2K_1}-1\right)m_i\left(z_i - rp_i\right)^2}{\left(\overline{z}_i - rp_i\right)\left(z_i - rp_i\right) + \left(e^{2K_2}-1\right)m_i\left(z_i - rp_i\right)^2} > 1.$$

Since prices are unchanged, $\left(\overline{z}_i - rp_i\right)\left(z_i - rp_i\right)$ and $m_i\left(z_i - rp_i\right)^2$ are unchanged. Since $K_1' > K_1$ and $K_2' < K_2$, the second term in π_{1i} increases and the second term in π_{2i} decreases. □

Proof of Proposition 5. **(i)** Using equilibrium prices, $\overline{p}_i = \frac{1}{r}\left(\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{1+\phi m_i}\right)$. Per Lemma 4, ϕm_i is increasing in ϕ. Hence, for $i \in \{1, ..., k\}$, $\overline{p}_i$ is increasing in ϕ. The result for equilibrium expected excess returns $E\left[z_i - r\overline{p}_i\right]$ follows.

(ii) Since $\lambda E\left[q_{1i}\right] + (1-\lambda)\,E\left[q_{2i}\right] = \bar{x}$, it is sufficient to show that for $i \in \{1, ..., k'\}$, $E\left[q_{1i}\right]$ increases in response to symmetric capacity growth. Let $K \equiv K_1$, and $K_2 = \delta K$, with $\delta \in (0,1)$. Since

$E\left[q_{1i}\right] = \frac{1+m_i\left(e^{2K}-1\right)}{(1+\phi m_i)}\bar{x}$, then $\frac{dE[q_{1i}]}{dK} = \frac{\bar{x}}{(1+\phi m_i)^2}\left[\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK}\left(1+\phi m_i\right) - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}m_i\left(e^{2K}-1\right)\right]$.

Hence $sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}\frac{m_i\left(e^{2K}-1\right)}{1+\phi m_i}\right)$.

In the proof of Lemma 4, we show that $\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > 2e^{2K}\frac{d(\phi m_i)}{d\phi} > 0$. Hence,

$$sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(2e^{2K} - \frac{d\phi}{dK}\frac{m_i\left(e^{2K}-1\right)}{1+\phi m_i}\right)$$

$$= sign\left(2e^{2K} - \frac{2m_i\left[\lambda e^{2K}+(1-\lambda)\delta e^{2K\delta}\right]\left(e^{2K}-1\right)}{1+m_i\left[\lambda(e^{2K}-1)+(1-\lambda)\left(e^{2K\delta}-1\right)\right]}\right)$$

$$= sign\left(e^{2K} - \left(e^{2K}-1\right)\frac{m_i\left[\lambda e^{2K}+(1-\lambda)\delta e^{2K\delta}\right]}{1+m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]-m_i}\right)$$

$$\overset{(1)}{=} sign\left(e^{2K} - \left(e^{2K}-1\right)\left[\frac{m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]}{1+m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]-m_i}\right]\right)$$

$$\overset{(2)}{=} sign\left(e^{2K} - \left(e^{2K}-1\right)\right) > 0$$

where (1) follows from $\delta \in (0,1)$, and (2) follows from the fact that the term in square brackets is less than 1.

(iii) Let the per capita capital income be decomposed into a component C_i that is common across investor groups, and a component that is group-specific:
$\pi_{1i} = c_i + \frac{1}{\rho\sigma_i^2}m_i\left(e^{2K}-1\right)(z_i - rp_i)^2$, where $c_i \equiv \frac{1}{\rho\sigma_i^2}(\overline{z} - rp_i)(z_i - rp_i)$, with expected value C_i.
Then $E[\pi_{1i}] = C_i + \frac{1}{\rho\sigma_i^2}m_i\left(e^{2K}-1\right)E\left[(z_i - rp_i)^2\right] = C_i + \frac{1}{\rho}m_i\left(e^{2K}-1\right)G_i$, where G_i is the gain from learning about asset i, equated across all $i \in \{1, ..., k\}$.
We then obtain that $\frac{E[\pi_{1i}]}{E[\pi_{2i}]} = \frac{C_i+\frac{1}{\rho}m_i\left(e^{2K}-1\right)G_i}{C_i+\frac{1}{\rho}m_i\left(e^{2K\delta}-1\right)G_i}$.
In response to an increase in K, C_i and G_i decrease, but they affect both sophisticated and unsophisticated profits in the same way. From Lemma 4, $m_i\left(e^{2K}-1\right)$ increases by more than $m_i\left(e^{2K\delta}-1\right)$ in response to a change in K. Hence overall, $\frac{E[\pi_{1i}]}{E[\pi_{2i}]}$ increases.

(iv) The volume of asset i is

$V_i = \frac{2m_i}{\sqrt{\pi}}\left[\frac{\lambda\sqrt{e^{2K}-1}+(1-\lambda)\sqrt{e^{2K\delta}-1}}{\rho\sigma_i}\right]$.

In Lemma 4, we show that for $i \in \{\overline{\imath}, ..., k\}$, $\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > \frac{d\left[m_i\left(e^{2K\delta}-1\right)\right]}{dK} > 0$, hence V_i increases. For assets $i \in \{1, ..., \overline{\imath}-1\}$, $\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > 0$, while $m_i\left(e^{2K\delta}-1\right)$ may increase or decrease, depending on δ. Moreover, for $i \in \{k+1, ..., k'\}$ (that is, for newly actively traded assets), volume increases from 0. In the proof of Lemma 4, we show that the decline in $m_i\left(e^{2K\delta}-1\right)$ is bounded by a term that is smaller than the bound on the increase in $m_i\left(e^{2K\delta}-1\right)$. Hence on net, V_i increases for $i \in \{1, ..., \overline{\imath}-1\}$ as well. Therefore, total volume, $V' = \sum_{i=1}^{k'} V_i' > \sum_{i=1}^{k} V_i = V$. Hence turnover $T \equiv V'/n\overline{x}$ increases. □

Proof of Proposition 6. We consider the choice of an individual investor, taking the choices of all other investors as given, characterized by the solution in the main text.

Case A. First, we consider the case in which the investor treats the price as any other random variable that cannot be processed perfectly for free. Suppose that the investor allocates capacity to learning the price of asset i. This investor will observe a compressed representation of the price, s^p_{ji}, that is the result of the decomposition $p_i = s^p_{ji} + \varepsilon_{ji}$,with $s^p_{ji} \sim \mathcal{N}\left(\overline{p}_i, \sigma^2_{spji}\right)$, $\varepsilon_{ji} \sim \mathcal{N}\left(0, \sigma^2_{\varepsilon ji}\right)$, and $\sigma^2_{pi} = \sigma^2_{spji} + \sigma^2_{\varepsilon ji}$. The amount of capacity consumed by the price signal is

$$I\left(p_i; s^p_{ji}\right) = \tfrac{1}{2}\log\left(\tfrac{\sigma^2_{pi}}{\sigma^2_{\varepsilon ji}}\right).$$

The quantity of information about *payoffs* that is conveyed by the price signal is

$$I\left(z_i; s^p_{ji}\right) = H\left(z_i\right) + H\left(s^p_{ji}\right) - H\left(z_i, s^p_{ji}\right) = \tfrac{1}{2}\log\left(\tfrac{\sigma^2_i \sigma^2_{spji}}{\left|\Sigma_{z_i s p_{ji}}\right|}\right),$$

where $\left|\Sigma_{z_i s p_{ji}}\right|$ is the determinant of the variance-covariance matrix of z_i and s^p_{ji}. Using the fact that z_i and s^p_{ji} are conditionally independent given prices, $Cov\left(z_i, s^p_{ji}\right) = Cov\left(z_i, p_i\right) Cov\left(p_i, s^p_{ji}\right)/\sigma^2_{pi}$. Using the solution for equilibrium prices, $Cov\left(z_i, p_i\right) = b_i\sigma^2_i$. Using the signal structure, $Cov\left(p_i, s^p_{ji}\right) = \sigma^2_{spji}$. Hence $Cov\left(z_i, s^p_{ji}\right) = b_i\sigma^2_i\sigma^2_{spji}/\sigma^2_{pi}$. The determinant becomes

$$\left|\Sigma_{z_i s p_{ji}}\right| = \sigma^2_i \sigma^2_{spji}\left(\tfrac{\sigma^2_{pi}\sigma^2_{pi} - b^2_i\sigma^2_i\sigma^2_{spji}}{\sigma^2_{pi}\sigma^2_{pi}}\right), \text{ so that}$$

$$I\left(z_i; s^p_{ji}\right) = \tfrac{1}{2}\log\left(\frac{\sigma^2_{pi}}{c^2_i\sigma^2_{xi} + \frac{b^2_i\sigma^2_i}{\sigma^2_{pi}}\sigma^2_{\varepsilon ji}}\right).$$

Next, we show that $I\left(z_i; s^p_{ji}\right) \leq I\left(p_i; s^p_{ji}\right)$. Suppose not. Then, in order for the reverse inequality to hold, it must be the case that

$$c^2_i\sigma^2_{xi} < \left(1 - \tfrac{b^2_i\sigma^2_i}{\sigma^2_{pi}}\right)\sigma^2_{\varepsilon ji} \quad \Leftrightarrow \quad \sigma^2_{pi} < \sigma^2_{\varepsilon ji},$$

which is a contradiction. Hence, $I\left(z_i; s^p_{ji}\right) \leq I\left(p_i; s^p_{ji}\right)$, with equality if and only if $\sigma^2_{pi} = \sigma^2_{\varepsilon ji}$, which occurs only if $I\left(p_i; s^p_{ji}\right) = 0$. Hence for any positive capacity dedicated to the price signal, the effective amount of information about the payoff is less than the capacity consumed in order to receive the signal.

Case B. Next, we consider the case in which the price itself is a perfectly observed signal that nonetheless consumes capacity. Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then $y_i \sim \mathcal{N}\left(\overline{y}_i, \sigma^2_{yi}\right)$, with

$$\overline{y}_i = \sigma^2_{yi}\left[\tfrac{1}{\sigma^2_i}\overline{z}_i + \tfrac{b^2_i}{c^2_i\sigma^2_{xi}}z_i - \tfrac{b_i}{c_i\sigma^2_{xi}}\left(x_i - \overline{x}_i\right)\right]$$

$$\tfrac{1}{\sigma^2_{yi}} = \frac{1}{\frac{1}{\sigma^2_i} + \frac{b^2_i}{c^2_i\sigma^2_{xi}}}.$$

The information contained in the price of asset i is $I(z_i;p_i) = \frac{1}{2}\log\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)$. Using the solution for equilibrium prices, this variance is given by

$\sigma_{yi}^2 = \frac{\sigma_i^2}{1+\left(\frac{\phi m_i}{\rho\sigma_i\sigma_{xi}}\right)^2}$.

We next demonstrate that the investor's ex-ante expected utility is higher when allocating all her capacity to learning from private signals than when allocating at least a portion of her capacity to learning from prices, owing to strategic substitutability. The investor's objective is to maximize

$$\widetilde{E}_{1j}\left[U_{2j}\right] = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widetilde{V}_{ji}+\widetilde{R}_{ji}^2}{\widetilde{\sigma}_{ji}^2}\right) \ s.t. \ \prod_{i=1}^{n}\left(\frac{\sigma_i^2}{\widetilde{\sigma}_{ji}^2}\right) \leq e^{2K_j},$$

where $\widetilde{R}_{ji}$ and $\widetilde{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $(\widetilde{\mu}_{ji} - rp_i)$, $\widetilde{\mu}_{ji}$ and $\widetilde{\sigma}_{ji}^2$ denote the mean and variance of the investor's posterior beliefs about the payoff z_i, and the tilde indicates that these variables are computed under a signalling mechanism that allows for learning from prices.

Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then, the investor designs a signal conditional on the information obtained from the price, $y_i = \widetilde{s}_{ji} + \widetilde{\delta}_{ji}$, where we maintain the same two independence assumptions that were used in setting up the private signal in the absence of learning from the price. Under this signal structure, the ex-ante mean is the same, regardless of whether the investor learns from p_i or not: $\widetilde{R}_{ji} = \overline{z}_i - r\overline{p}_i$. The ex-ante variance of expected excess returns is given by $\widetilde{V}_{ji} = Var_{1j}\left(\widetilde{\mu}_{ji}\right) + r^2\sigma_{pi}^2 - 2rCov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right)$. Using the formula for partial correlation and exploiting the fact that signals and prices are conditionally independent given beliefs, $Cov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right) = Cov_{1j}\left(\widetilde{\mu}_{ji}, y_i\right)Cov_{1j}\left(y_i, p_i\right)/\sigma_{yi}^2$. Using the signal structure, $Cov_{1j}\left(\widetilde{\mu}_{ji}, y_i\right) = Var\left(\widetilde{s}_{ji}\right)$, $Var\left(\widetilde{s}_{ji}\right) = \sigma_{yi}^2 - \widetilde{\sigma}_{ji}^2$, and using equilibrium prices, $Cov_{1j}\left(y_i, p_i\right) = b_i\sigma_i^2$. Hence $Cov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right) = b_i\sigma_i^2 - b_i\sigma_i^2\widetilde{\sigma}_{ji}^2/\sigma_{yi}^2$. Hence $\widetilde{V}_{ji} = \left(1-2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2 - \left(\sigma_i^2 - \sigma_{yi}^2\right) - \left[1 - 2rb_i\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)\right]\widetilde{\sigma}_{ji}^2$, if the investor learns from p_i.

Conversely, if the investor does not allocate any capacity to learning from prices, $V_{ji} = \left(1-2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2 - \left(1-2rb_i\right)\widetilde{\sigma}_{ji}^2$, where we have used the fact that the information constraint implies that the investor's posterior variance, here denoted by $\widetilde{\sigma}_{ji}^2$, is the same in both cases. Both cases imply a corner solution, with the investor allocating all capacity to learning about a single asset. The remaining question is: will the investor allocate any capacity to learning from the price, or will she use all capacity on the private signal? It can be easily seen that for any positive level of capacity allocated to the price signal, $V_{ji} > \widetilde{V}_{ji}$. Hence, the investor's ex-ante utility is lower when she devotes any positive amount of capacity to learning from prices. Learning from prices increases the covariance between the investor's posterior beliefs and equilibrium prices, thereby reducing the investor's excess returns. This case is similar to that of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013), who show that prices are an inferior source of information in a portfolio choice model with an additive constraint on the sum of signal precisions.

Hence, regardless of the informativeness of prices relative to the investor's capacity, the investor is always better off learning through signals that provide information directly on the payoffs. In our framework prices lose their special role as publicly available signals. $\square$

Example with endogenous capacity choice

Below, we provide a numerical example of an endogenous capacity choice outcome in a model in which wealth heterogeneity matters for endogenous capacity choice. In particular, we assume that investors have identical CRRA preferences with IES coefficient γ, and differ in terms of their beginning of period wealth. Then, for each investor j, the absolute risk aversion coefficient is a function of wealth W_j, given by

$$A(W_j) = \gamma / W_j.$$

Locally, we map this into absolute risk aversion differences in a mean-variance optimization model by setting the coefficient ρ_j for investor j equal to $A(W_j)$. What these differences in absolute risk aversion imply in the model is differences in the size of the risky portfolio, and hence different gains from investing wealth in purchases of information capacity.

In particular, for a given cost of capacity given by the function $f(K)$, each investor type is going to choose the amount of capacity to maximize the ex-ante expectation of utility:

$$\frac{1}{2\rho_j} \sum_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ij}^2} G_i - f(K_j),$$

where, in equilibrium, G_i is a function of the distribution of individual capacity choices of investors, but not of individual capacity choices, and $\widehat{\sigma}_{ij}^2 = \sigma_i^2 e^{-2K_j}$ if the investor learns about asset i.

The gain from increasing is given by the benefit of increasing the precision of information for the asset that the investor is learning about. Since all actively traded assets have the same gain in equilibrium, we can express the marginal benefit of increasing capacity in terms of the gain of the highest volatility asset (asset 1), $\frac{1}{2\rho_j} e^{2K_j} G_1$, and then the optimization problem for capacity choice can be expressed as

$$\max_K \left\{ \frac{1}{2\rho_j} e^{2K} G_1 - f(K) \right\}. \tag{22}$$

Assumption 3 below ensures an interior solution to (22) exists.

Assumption 3. The following statements hold:
(i) For all j, $\frac{G_1}{\rho_j} - f'(0) > 0$, where G_1 is evaluated at $K_j = 0$ for all j,
(ii) There exists $\underline{K} > 0$, such that for all j and for all $K > \underline{K}$, $2\frac{G_1}{\rho_j} e^{2K} - f''(K) < 0$,
(iii) There exists $\bar{K} > 0$ such that for all j and for all $K > \bar{K}$, $\frac{G_1}{\rho_j} e^{2K} - f'(K) < 0$.

Numerical example Assume that the cost function is of the form: $f(K) = e^{aK}$. Under assumption 3, the optimal choice of K for agent j is implicitly defined by:

$$\frac{G_1(\{\bar{K}_j\})}{\rho_j} = a e^{(a-2)K},$$

where we make the dependence of G_1 on the distribution of capacities explicit. Clearly, for any $a > 2$, the higher wealth investors (implying lower ρ_j) will choose higher capacity levels. However, because of the dependence of G on equilibrium capacity choices, to quantify the differences we need to solve the equilibrium fixed point of the model.

Figure 10 presents the ratio of capacities as a function of the cost parameter of capacity, a, for different values of the absolute risk aversion coefficient of the wealthy ρ_1 (which maps into different common relative risk aversion coefficients γ). The inequality in capacity exhibits a U-shape. First, if the cost of capacity is small, then the equilibrium inequality in capacity grows without bound, as the wealthier accumulate infinite capacity (faster than the less wealthy). For higher values of the cost of capacity, inequality exhibits a growing trend as the cost increases, very quickly approaching values in excess of 38, our benchmark value. It should be noted that even for the high values of the cost parameter, the overall cost relative to gain, $f(K_j)/\frac{1}{2\rho_j}e^{2K_j}G_1$, is relatively small, less than 1% for the wealthy and less than 6% for the less wealthy.

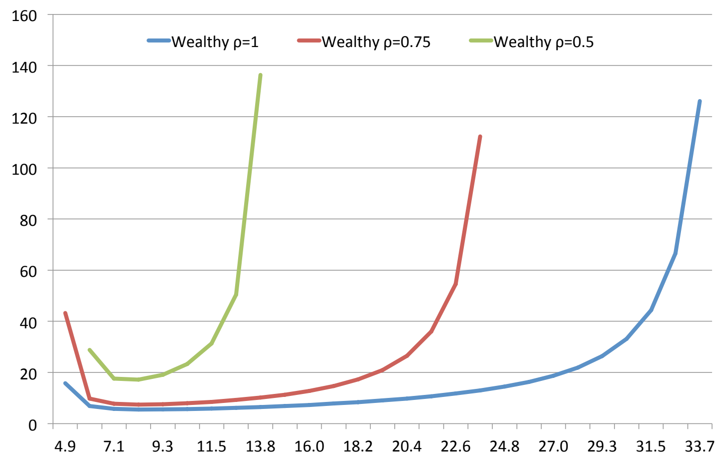

Figure 10: Inequality in information capacity (K_1/K_2) as a function of a and wealthy absolute risk aversion coefficient.

CRRA Utility Specification

Here, we solve the main investment problem of maximizing the expected utility of wealth, where the utility function is CRRA with respect to end of period wealth:

$$\max E\frac{W^{1-\rho}}{1-\rho} \tag{23}$$

where $\rho \neq 1$. Generally, for our specification of the payoff process, i.e. $z \sim \mathcal{N}(\bar{z}, \sigma_i^2)$, wealth next period is

$$W_{t+1} = r(W_t - \sum_i p_i q_i) + \sum_i q_i z_i$$

which has a normal distribution if z_i's are normal. In order to analytically express the expectation in (23), we start by expressing wealth as $W' = We^{\log\{[r(1-\sum p\frac{q}{W})+\sum \frac{q}{W}z]\}}$, and then use an approximation of the log of return.

Approximation To approximate $\log\{[r(1-\sum p\frac{q}{W})+\sum \frac{q}{W}z]\}$, define

$$f(z-rp) \equiv \log[r + \frac{1}{W}\sum pq\frac{z-rp}{p}].$$

In the above equation, the term z is the only unknown stochastic term. The Taylor approximation is

$$f(z-rp)=f(\bar{z}-rp)+f'(\bar{z}-rp)(z-\bar{z})+\frac{1}{2}f''(\bar{z}-rp)(z-\bar{z})^2+o(z-rp)$$

where in the above,

$$f'=\frac{1}{r+\frac{1}{W}\sum q(\bar{z}-rp)}\frac{q}{W},$$

$$f''=-\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^2}\frac{q^2}{W^2},$$

$$f'''=2\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^3}\frac{q^3}{W^3}.$$

With these formulas in hand, the approximation is

$$f(z-rp)=\log[r+\frac{1}{W}\sum q(\bar{z}-rp)]+\frac{1}{r+\frac{1}{W}\sum q(\bar{z}-rp)}\frac{q}{W}(z-\bar{z})$$

$$-\frac{1}{2}\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^2}\frac{q^2}{W^2}(z-\bar{z})^2$$

Denote

$$r+\frac{1}{W}\sum q(\bar{z}-rp)\equiv R(q)$$

Then we can write

$$f(z-rp)=\log[R(q)]+\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}\frac{1}{R(q)^2}\frac{q^2}{W^2}(z-\bar{z})^2,$$

and

$$(e^{\log(f(z-rp))})^{1-\rho}=e^{(1-\rho)(\log[R(q)]+\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2)}$$

$$=(R(q))^{1-\rho}e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2}$$

We are interested in the object $e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2}$ from the above expression. First, we approximate the term $(z-\bar{z})^2$ by its expected volatility, $\sigma^2_{\delta i}$, to get

$$e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma^2_{\delta i}}$$

As an approximation point, we pick $\bar{z}$, which gives a constant $R(q)$, and then

$$\log EW^{1-\rho}=const.\times\log Ee^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma^2_{\delta i}} \tag{24}$$

where the variable in the exponent is normal, with mean (ignoring constants) $\sum q_i(\hat{\mu}_i-\bar{z}_i)$ and variance equal to $\sum q_i^2\sigma^2_{\delta i}$. Then,

$$\log EW^{1-\rho}=const.\times(1-\rho)\left\{\frac{1}{R}\sum\frac{q}{W}(\hat{\mu}_i-\bar{z}_i)+(1-\rho)\frac{1}{W^2R^2}\frac{1}{2}\sum q_i^2\sigma^2_{\delta i}\right.$$

$$-\frac{1}{2}\frac{1}{W^2R^2}\sum q_i^2\sigma_{\delta i}^2\Big\}$$

which gives

$$\log EW^{1-\rho} = const. \times (1-\rho)\left\{\frac{1}{R}\sum\frac{q}{W}(\hat{\mu}_i - \bar{z}_i) - \rho\frac{1}{W^2R^2}\frac{1}{2}\sum q_i^2\sigma_{\delta i}^2\right\}$$

Interior minimum (which maximizes $EW^{1-\rho}/(1-\rho)$) is

$$q_i = \frac{1}{\rho}\frac{\hat{\mu}_i - rp}{\sigma_{\delta i}^2}(Wr).$$

Plugging in gives:

$$U = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}e^{\frac{1-\rho}{\rho}\frac{1}{2}\sum\frac{(\hat{\mu}_i - rp)^2}{\sigma_{\delta i}^2}}$$

where $\hat{\mu}_i$ and $\sigma_{\delta i}$ are the expected mean and standard deviation of the payoff process z, given the investor's prior, private signal and the price signal.
As in Brunnermeier (2001) to compute the expectation E(U). Some new notation is needed for that. First, denote the excess return as

$$R_i \equiv \hat{\mu}_i - rp_i$$

with mean $\hat{R}_i$. Denote the period zero volatility of $R_i - \hat{R}_i$ as $\hat{V}_i$ (which is just the volatility of R_i). Then we can write (in matrix form):

$$U = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}e^{\frac{1-\rho}{\rho}\frac{1}{2}[(R-\hat{R})\Sigma_\delta^{-1}(R-\hat{R}) + 2\hat{R}\Sigma_\delta^{-1}(R-\hat{R}) + \hat{R}\Sigma_\delta^{-1}\hat{R}]}$$

Which gives

$$EU = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}|I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1}|^{-1/2}\times$$

$$\exp(\frac{(1-\rho)^2}{2\rho^2}\hat{R}\Sigma_\delta^{-1}(I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1})^{-1}\hat{V}\hat{R}\Sigma_\delta^{-1} + \frac{1-\rho}{2\rho}\hat{R}\Sigma_\delta^{-1}\hat{R})$$

which becomes

$$EU = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}(\Pi_i(1 - \hat{V}_i\frac{1-\rho}{\rho}\sigma_{\delta i}^{-1}))^{-1/2} \times \exp\left(\frac{1-\rho}{2\rho}\sum\frac{\hat{R}_i^2}{\sigma_{\delta i}}\left[(1 + \frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho})^{-1}\right]\right)$$

logging the negative of that and simplifying gives

$$-\log(-EU) = const. + \frac{1}{2}\sum_i \log(1 + \frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho}) + \frac{\rho-1}{2\rho}\sum_i\frac{\hat{R}_i^2}{\sigma_{\delta i} + \hat{V}_i\frac{\rho-1}{\rho}}$$

This objective function is strictly decreasing in $\sigma_{\delta i}$ and convex, which means that agents are going to invest all capacity into learning about one asset. For that asset, $\sigma_{\delta i} = e^{-2K}\sigma_{yi}$, and $\sigma_{\delta i} = \sigma_{yi}$ otherwise.