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# Computer Simulation for Comparison of Proposed Mechanisms for FCC Incentive Auction

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By: Justin Huang
Advisor: Professor Lawrence Ausubel
University of Maryland, College Park

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## Abstract

This paper examines the differences between the Simultaneous Multiple Round Auction (SMRA) and Clock Auction with Intra-round Bidding in the context of a computer-simulated FCC incentive auction. Bidders are assumed to bid in a straightforward manner, and n=15 sets of initial conditions are simulated on both of the proposed auction mechanisms. Analysis is performed to compare number of rounds, revenue generated, aggregate bidder utility, and possibility for undesirable strategic behavior. Final results show that the Clock Auction with Intra-round Bidding achieves significant reduction in number of rounds over the SMRA, generating more revenue but lower aggregate bidder utility. When examining social welfare (defined as the sum of revenue and aggregate bidder utility), we find no significant difference among the auctions. Additional findings highlight the importance of an Anti-Stalling Rule in ensuring straightforward bidding does not yield a near-infinite auction, and an extension of this scenario suggests that additional rules are needed to preclude the possibility of collusion by bidders to stall the auction indefinitely.

## 1. Introduction and Background

This paper presents the results of the simulated performance of two different auction types within the context of an FCC spectrum auction. Specifically, the Simultaneous Multiple Round Auction (SMRA) and Clock Auction with Intra-round Bidding are explored as possible implementations of the Forward Auction component of the FCC's upcoming incentive auction (see NPRM Oct 2), part of the FCC's Broadband Acceleration Initiative. The stated purpose of the FCC's incentive auction is to free and reallocate bandwidth currently used for TV broadcast towards wireless broadband usage (such as 4G LTE and Wi-Fi like networks). Per the NPRM, the proposed incentive auction has three components, the Reverse Auction, Repacking, and the Forward Auction. In the Reverse Auction, TV broadcasters are given the opportunity to voluntarily relinquish their current bandwidth, share a station, or move to a lower bandwidth in the reverse auction. Through a proposed clock auction mechanism, TV broadcasters are offered progressively lower amounts in order to relinquish their bandwidth – this process continues until a desired amount of bandwidth has been freed for reallocation. This is followed by Repacking, whereby remaining broadcasters may be reorganized so that they occupy a smaller portion of the ultra high frequency (UHF) band (and those who elected to do so are moved into the VHF band) and contiguous bandwidth is freed for sale in the Forward Auction. The Forward Auction will reallocate the newly freed bandwidth, and the proposed mechanism for doing so is an Ascending Clock Auction for multiple items with an allowance for Intra-round Bidding.

Previous literature suggests that the use of Intra-round Bidding can reduce the number of rounds as compared to SMRA formats while avoiding the inefficiency associated with undersell in a Clock Auction with a coarse price grid (Ausubel and Crampton 2004). However, the upcoming FCC incentive auction presents novel challenges in the number of licenses sold at once, and it would be useful to verify the theoretical and historical results computationally. The purpose of this paper is to examine the differences between this proposed Forward Auction mechanism, the Clock Auction with Intra-round Bidding, and the SMRA, which has traditionally been used by the FCC since 1994 for auctions of multiple units. Comparison is made using computer simulation, modeling straightforward bidder activity as firms compete for wireless licenses exhibiting both substitute and complement effects in a reduced version of the FCC incentive auction.

A notable aspect of the Forward Auction is that the amount of bandwidth for sale may not be known in advance. Bandwidth is freed for reallocation in the Reverse Auction, which may occur simultaneously with the Forward Auction. Since the FCC will be compensating TV broadcasters in the Reverse Auction before receiving revenue at the conclusion of the Forward Auction, minimizing the gap between the two is a priority for auction design. Additionally, from the perspective of the bidding wireless firms, a shorter Forward Auction would reduce both uncertainty and participation cost – especially given the enormous importance of obtaining desired licenses to long-term business viability. In this case, auction length is directly related to the number of auction rounds, as generally only 1-2 rounds can be conducted per hour. As an illustrative example, if 1 round is conducted per hour, a 120-round auction would take 3 full weeks to resolve. Thus it is essential to both have an estimate of number of rounds to auction conclusion given starting conditions and quantify round reduction between auction formats. This also highlights the value of simulation, as it is not immediately apparent how many rounds it might take for hundreds of licenses to be allocated to tens of bidders, or how sensitive round count is to a range of possible initial conditions. While round count is an important criterion for comparison, other considerations include efficiency (total bidder utility), revenue generated, and potential for undesirable strategic behavior. All of these are examined in this paper.

The paper is organized as follows: Section 2 describes the methodology, specifically the simulation implementation, construction of initial conditions, and assumptions; Section 3 details simulation results over n=15 sets of initial conditions, complete with charts and supporting analytics; Finally, Section 4 concludes the paper with a summary of results, recommendations, caveats, and future work.

## 2. Methodology

The two auction mechanisms for comparison included the SMRA and Clock Auction with Intra-round Bidding. All simulations were conducted on the same simplified test map (Figure 1). This test map included twenty-four Economic Areas (EAs), each with eight identical licenses available for a total of 192 licenses. The map organization was chosen to allow enough richness for complexity and competition while still permitting results to be calculated in a reasonable period of time. Our test map with twenty-four EAs and a total of 192 licenses expands upon our previous map, which had sixteen EAs and a total of forty-eight licenses. While we had originally planned to model the full 176 EAs available for the incentive auction, this proved too computationally intensive for the simulation to complete in a reasonable amount of time.

The test map's twenty-four EAs were organized into four Regions. A total of fifteen bidders were modeled with types Regional and National – each of the twelve Regional bidders receive utility (payoff) only for acquiring licenses in one of the specified Regions, while three National bidders receive utility for acquiring licenses across the entire map. Each bidder's payoff structure was designed such that licenses exhibited substitute and complement effects, with private independent uniformly distributed payoffs determined in advance. Licenses within an EA were perfect substitutes – each bidder had an independent private value associated with acquiring a license in a particular EA. Licenses within a Region were complementary – acquiring a license in every EA within a Region yielded additional payoff, with super-additive payoff for acquiring a second set of licenses in each EA of a Region and sub-additive payoff for each additional set of licenses. In the case of National-type bidders, additional bonuses could be earned for acquiring licenses in every EA of every Region. This bonus structure was set up to simulate the real-world incentives to achieve continuous coverage over a region of the US.

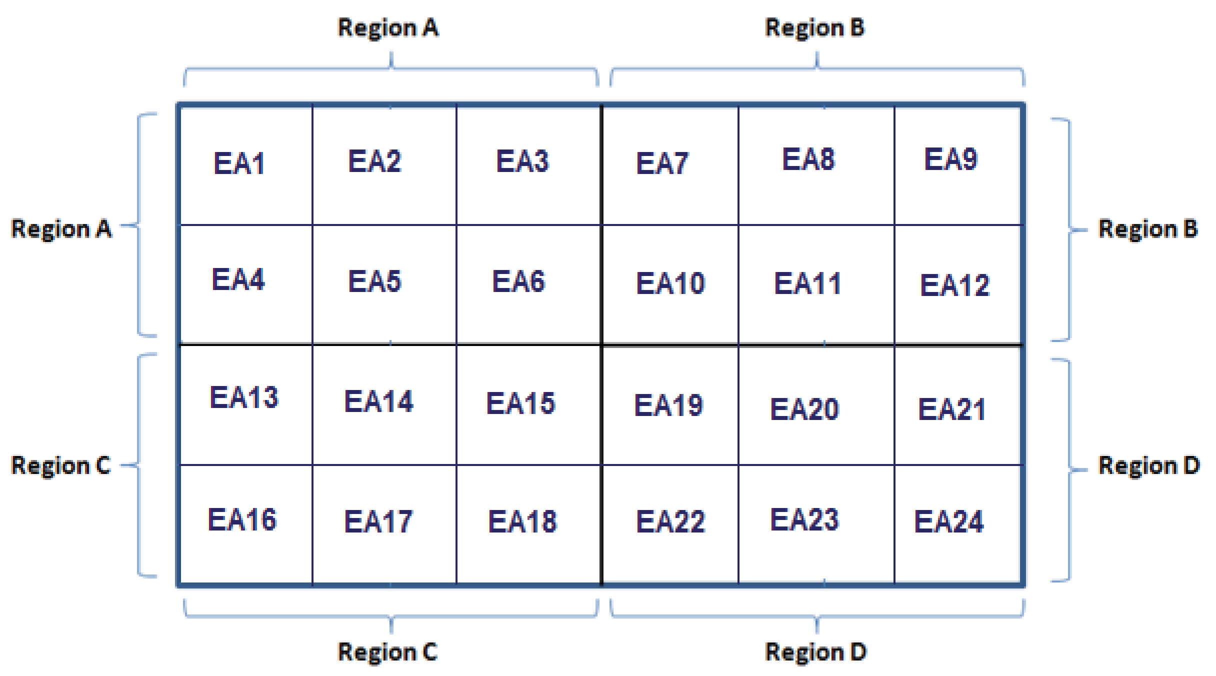


**Figure 1**: Twenty-four Economic Area Design with Four Regions

Twenty-four Economic Areas (EAs 1-24) are divided into four Regions (A-D), and each EA offers eight identical licenses. Twelve Regional-type bidders gain payoff for acquiring licenses in one of the four Regions, and three National-type bidders gain payoff for acquiring licenses across the entire map. Bonus payoff gained by acquiring a set of licenses for each EA in a Region, and National bidders earn additional payoff for acquiring a licenses in every EA on the map.

The entire simulation was modeled using MATLAB, a computational software package and language designed by MathWorks. MATLAB was chosen for its speed, widespread use for academic modeling, and integrability with Gurobi. Gurobi is an optimization software package for linear programming, quadratic programming, and mixed integer programming and provided a plugin that allowed its routines to be called directly from MATLAB. At the core of the model was the bidder logic. We assumed that bidders would place bids in a straightforward manner, solving the problem of choosing which licenses to bid on in order to maximize net payoff (given by the payoff structure described above minus prices of licenses acquired) given budget constraint and which licenses were provisionally won by the bidder (as none of the simulated auction formats allowed bids to be withdrawn). Mixed integer programming (MIP) provided the solution. The decision of whether or not to bid on a particular item was reduced to a binary variable. Additional binary variables associated were declared to detect when regional and national bonus payoff requirements were met and determine whether a bidder was locked into “bidding” for a license by virtue of being a provisional winner in a round. The equations described can be found in **Appendix B**.

Initial tests of our auction model showed inefficient allocations and underperformance of the Clock Auction with Intra-round Bidding as compared to the SMR Auction. Recent improvements to bidder logic improved Clock Auction license allocation and improved parity between the two auction types. Most notably, we introduce bid shading for bidders in both auction types. Bidders place their bids under the assumption that they will win the desired bid packages. If prices rise sufficiently or a bidder is outbid, a bidder may not achieve the expected synergy bonus but still may be locked into winning a number of licenses. Therefore, bid shading is necessary to account for this exposure risk. Through iterative experimentation, we determined that bid shading by approximately 40% of the synergy bonus is a Nash equilibrium strategy for all bidders. While updates to bidder logic (including bid shading and recognition of irreducible demand) did improve allocations, there is still room for improvement. For example, we expect that optimal bid shading would change as the auction progresses. An optimal bidder would greatly shade his bids while a number of other bidders remain in the auction and then decrease shading as other bidders drop out and allocations become more certain.

A second bidder improvement specifically applied to Clock Auction bidders. In previous implementations, simulated bidders could reduce their demand by an amount greater than excess demand during the process of Intra-round bidding. In the absence of increased demand from other bidders, some or all of this Intra-round bid (and any accompanying increases in demand on other licenses) would not be processed. The model's improved bidder logic restricts IR Clock Auction bidders from reducing their demand by an amount greater than the excess demand for the licenses in a particular EA, even though this bid could legally be submitted. This restriction on bidder logic could have the effect of slightly increasing the number of rounds to auction completion.

We constructed n=15 sets of initial conditions, including payoff structure (with independent uniformly distributed private valuations and regional/national bonuses), starting prices, and bidders' wealth. A sample initial condition set can be found in **Appendix D**. These same 15 sets of initial conditions were used across all simulated auction mechanisms to ensure comparability of results. Over these 15 runs of the simulation, we gathered results on number of rounds, aggregate bidder utility (payoff), and revenue generated. Trials were repeated for two different bid increments: 5% and 10%.

The Simultaneous Multiple Round Auction was modeled as follows: In each round, bidders are presented with current prices for each of the forty-eight available licenses (three licenses for each of the sixteen EAs). Bidders then submit bids in a straightforward manner, bidding the minimum increment (plus a small random delta to eliminate ties) if it would be utility maximizing for them to acquire that license given their budget constraint and the set of licenses provisionally won. At the end of the round, all bids are announced, and the highest bid for each license becomes the new provisional winner. Auction proceeds until a round passes where no new bids submitted, at which point the provisional winners on each license win the license and pay their high bid.

The Clock Auction with Intra-round Bidding was comparatively more complex to model: In the first round, bidders express their demand for licenses in each EA at the starting prices. In each successive round, if aggregate demand exceeds supply for licenses in an EA, posted end of round price for licenses in that EA increases by the bid increment. During each round, bidders express changes in demand at all points between start of round and end of round prices through the process of Intra-round Bidding. Each Intra-round bid specifies a change in a bidder's demand for licenses across any number of EAs at a particular percentage point between the start and end of round price. For an illustration of Intra-round Bidding, please see **Appendix C**.

After all Intra-round bids have been submitted, they are evaluated from lowest percentage point to highest. An Intra-round bid will be evaluated and then removed from the queue if the change in demand does not cause aggregate demand to fall below aggregate supply and if it complies with the Anti-Stall and other applicable rules. Once an Intra-round bid is accepted, all remaining Intra-round bids in the queue are examined again for feasibility at the new demand. Intra-round bidding ends when no more Intra-round bids can be accepted. At this point, if end of round aggregate demand in all EAs is not less than supply, the auction proceeds to the next round in the same manner as a standard Clock Auction. If aggregate demand falls below supply over the course of the round, Intra-round bids are examined to determine the price point at which demand for licenses in an EA fell equal to supply. The Clock Auction with Intra-round Bidding closes when aggregate demand equals supply for licenses in every EA, at which point the bidders are allocated licenses according to their demand and each pays a uniform price for licenses in an EA.

## 3. Simulation Results

We compare the results of n=15 simulated trials of the SMRA and Clock Auction with Intra-round Bidding across three attributes, number of rounds, aggregate bidder utility, revenue generated, and social welfare (defined as the sum of revenue generated and aggregate bidder utility). The same sets of initial conditions are used for each auction type and trials are repeated for both a 5% and 10% minimum bid increment. A sample Clock Auction with Intra-round Bidding simulation output can be found in **Appendix E** and results of the simulations are summarized in **Table 1**.

| | **SMRA** 5% Bid Increment | **SMRA** 10% Bid Increment | **Clock Auction with Intra-round Bidding** 5% Bid Increment | **Clock Auction with Intra-round Bidding** 10% Bid Increment |
|---|---|---|---|---|
| Number of Rounds | 59.47<br>(10.82) | 25.13<br>(3.69) | 30.53<br>(1.43) | 15.80<br>(1.22) |
| Aggregate Bidder Utility | 827.66<br>(12.92) | 825.38<br>(11.90) | 813.08<br>(11.70) | 806.19<br>(12.39) |
| Revenue | 734.99<br>(14.19) | 716.92<br>(11.28) | 741.38<br>(9.70) | 755.85<br>(9.89) |
| Social Welfare (Revenue+Bidder Utility) | 1562.65<br>(20.78) | 1542.30<br>(21.85) | 1554.46<br>(19.09) | 1562.04<br>(22.51) |

**Table 1:** Summary Statistics by Auction Type
The table displays the results of n=15 simulations of each of the auction types (with corresponding bid increment) given the same sets of initial conditions. Means in each category are displayed with sample standard deviations below in parenthesis.

From the table above, the most notable result is in round reduction. The Clock Auction with Intra-round Bidding reduces the number of rounds to auction completion by roughly 40% as compared to the SMRA. This result holds across both bid increments and confirms theoretical and previous historical results. Round reduction is to be expected given the structure of the auctions – when excess demand exists across the licenses of multiple EAs, prices for the licenses rise simultaneously by the bid increment in each round. In comparison, the SMRA format would have bidders alternate in bidding up various licenses while maintaining the high bid on items provisionally won, slowing the price discovery process. For both bid increments, the round reduction is significant using a standard t-test at the .01 level. A graphical representation of the round reduction can be seen in **Figure 2**.

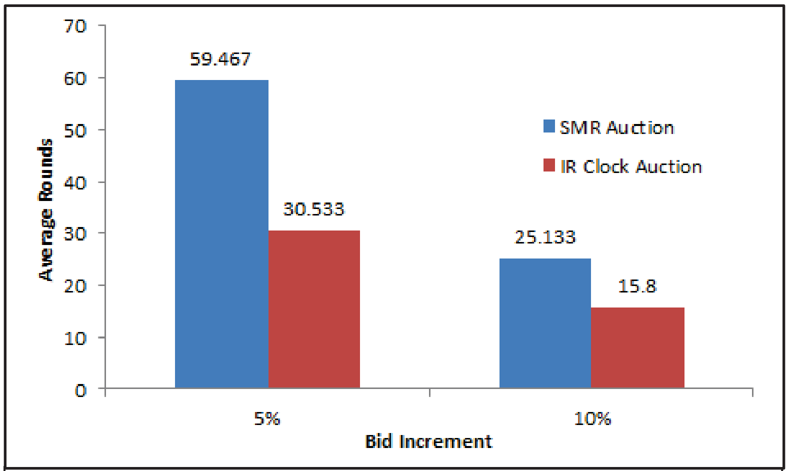


**Figure 2:** Rounds Comparison of SMRA and Clock Auction with Intra-round Bidding
The above bar graph displays the average number of rounds for auction completion across two auction types and two bid increments.

Another consideration is whether aggregate bidder utility and revenue generated are comparable between the Clock Auction with Intra-round Bidding and the SMRA. Simulation showed that using the Clock Auction with Intra-round Bidding decreased aggregate bidder utility by 1.76% under a 5% bid increment rule and 2.32% when a 10% under a bid increment rule. The Clock Auction with Intra-round Bidding generated .87% and 5.43% more revenue at 5% and 10% bid increments, respectively. The combination of revenue increase and bidder utility decrease had additive implications for bidder surplus, which the Clock Auction with Intra-round Bidding reduced by 22.6% and 53.6% at the two bid increments. Interestingly, an increase in bid increment is seen to decrease revenue for the SMR Auction while increasing revenue for the IR Clock Auction. This result could be examined in future simulations or verified experimentally. The differences in bidder utility (at both bid increments) and the difference in revenue at the 10% bid increment between the Clock and SMR auctions were significant using a standard t-test at the .05 level. If we define social welfare as the sum of bidder utility and revenue generated, we see that the difference between the four auction formats is minimal – a maximum of 1.3% (not significant at the .05 level). These results are given graphically by **Figure 3** (above) and **Figure 4** (below).

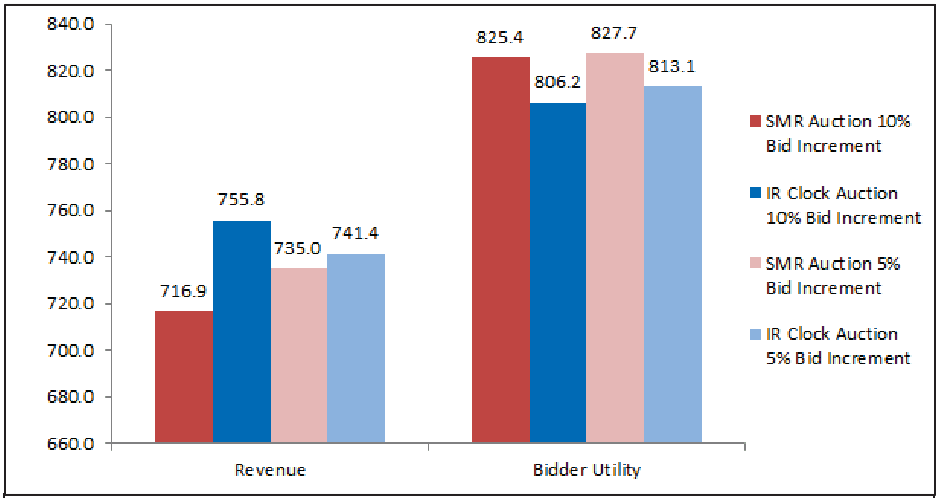


**Figure 3**: Aggregate Bidder Utility and Revenue Comparison
Bidder utility and revenue averages are shown in the above bar graph, displaying results from the two auction types at the 5% and 10% bid increment.

In addition to the results enumerated above on number of rounds, bidder utility, revenue, and social welfare, our computer simulation provided an interesting insight into stalling and strategic behavior. The a previous version of the anti-stall rule (ASR) "prohibits bidders from both increasing and decreasing demand for the same Product at different price points in the same round, or from increasing demand for a Product in a round where its price did not change and then reducing demand for the same Product in the next succeeding round." (Incentive Auction Rules Option and Discussion Oct 2012) We find was that, in the absence of the Anti-Stalling Rule, a straightforward and rational bidder would frequently shift demand at a fraction of end of round prices, causing rounds to end without significant price movement. Even with the Anti-Stall Rule as stated above, the auction could be stalled indefinitely in a similar manner if a second bidder colluded to reduce demand during the period when the first was locked in by the ASR (thereby keeping price from rising significantly). An illustration of this stalling cycle is given in **Figure 5**.

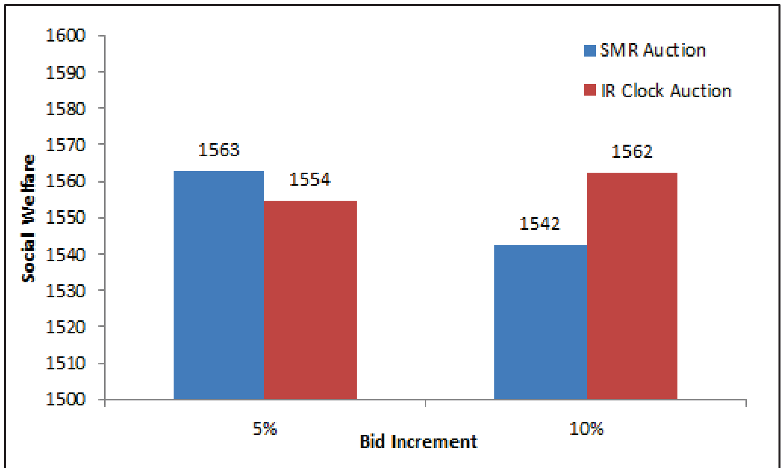


**Figure 4**: Social Welfare Comparison
Social Welfare (defined as the sum of bidder utility and revenue generated) is shown in the above bar graph, with results from the two auction types at the 5% and 10% bid

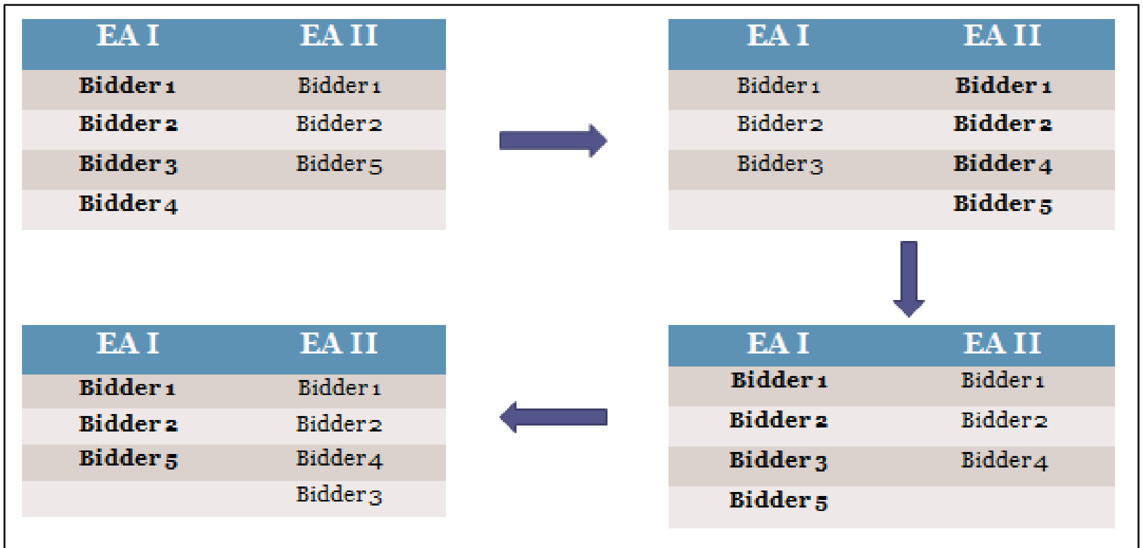


**Figure 5**: Stalling Cycle
Even with the Anti-Stall Rule as quoted above, a cycle of bidders could stall the auction. In this example, three licenses are available in each EA. Bidders 1 and 2 demand one license in both of the EAs, while bidders 3, 4, and 5 demand one license in either of the EAs. Since end of round price is only incremented when there is excess demand, a cycle of bidders could switch their demands to the product without excess demand to stall the auction.

One proposed solution is that end of round prices may be specified as non-decreasing. Given this constraint, end of round prices in period T+1 will not be lower than end of round prices in period T, regardless of whether demand fell equal to supply for licenses in a particular EA. A second solution would be for end of round prices to be strictly greater than start of round prices even when there is no excess demand. Both of these mechanisms can be shown to prevent the stalling cycle illustrated above. The prevention of stalling is crucial to FCC incentive auction design, as rarely are there provisions that would allow the seller to break stalls or by introducing or modifying rules mid-auction.

## 4. Conclusion

Based on our simulation results, we find that the Clock Auction with Intra-round Bidding provides a compelling alternative to the SMRA for multiple unit auctions of items exhibiting complement and substitution effects, such as in the context of FCC incentive auctions. Most notably, the Clock Auction with Intra-round Bidding reduced the number of rounds to auction conclusion by up to 50%. We find that the effect of auction type on total social welfare is not significant, though the distribution of profits between buyers and sellers is sensitive to exact bidder behavior. The Clock Auction with Intra-round Bidding seems to favor the seller, increasing revenues by .87-5.43% on average. From the bidder perspective, aggregate utility decreased 1.7-2.3% on average and the combination of decreased utility and increased prices (revenue for seller) greatly diminished bidder surplus. In the process of building and running our computer simulation, we highlighted the importance of the Anti-Stalling Rule in ensuring straightforward bidding does not yield a near-infinite auction, and an extension of this scenario suggests that changes to End of Round Pricing are needed to preclude the possibility of collusion by bidders to stall the auction indefinitely.

While results are promising, creating this simulation was not without difficulties and challenges. Some of the limitations of our simulation stem from the use of linear optimization to drive bidder logic. Certain higher order processes such as evaluating past bidder behavior or incorporating expectations were difficult to express in a manner that can be evaluated using MIP. It should be noted that our simulation was run on a limited test map with less complexity than will exist within the actual FCC incentive auction. It was expected that bid increment would have a greater effect on aggregate bidder utility than we observed in the SMRA simulation, suggesting that tweaks to the payoff structure may be needed. As previously mentioned, dynamic bid shading could bring simulated bidder logic closer to expected real bidder behavior. We recommend that future simulations closely examine the computational method by which to evaluate optimal bids, feature the suggested model improvements, and evaluate a greater number of bid increments.

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## Appendix B: Mixed Integer Programming

The following equations define the Mixed Integer Programming problem solved by each bidder to determine optimal bids in each round.

Variable Declarations

$\boldsymbol{x_i}$: *Binary decision variable to determine whether a bidder will bid on license i*

$\boldsymbol{cw_i}$: *Binary variable denoting whether bidder is currently(provisionally)winning license i and therefore cannot withdraw bid*

$\boldsymbol{\alpha_{k,j}}$: *Dummy binary variable denoting whether bidder wins at least j licenses in EA k*

$\boldsymbol{y_{r,j}}$: *Binary decision variable representing whether bidder wins the* $j^{th}$ *complete set of licenses in every EA of Region r*

$\boldsymbol{z_{N,j}}$: *Binary decision variable representing whether bidder wins the* $j^{th}$ *complete set of licenses in every EA on the test map (N denotes "national")*

$\boldsymbol{U_i}$: *Utility granted to bidder for with winning license i*

$\boldsymbol{B_{r,j}}$: *Utility bonus granted to bidder for winning the* $j^{th}$ *complete set of licenses in every EA in Region r*

$\boldsymbol{B_{N,j}}$: *Utility bonus granted to bidder for winning the* $j^{th}$ *complete set of licenses in every EA on the test map*

$\boldsymbol{p_i}$: *Effective price of license i for this MIP*

$\boldsymbol{W}$: *Bidder wealth*

$\boldsymbol{\pi}$: *Bidder profit*

Objective Function (to be maximized)

$$\boldsymbol{\pi} = \sum_i x_i * (U_i - p_i) \quad + \quad \sum_{j,r} y_{j,r} * B_{j,r} \quad + \quad \sum_j z_{N,j} * B_{N,j}$$

Constraints

Budget:

$$\sum_i x_i * p_i \leq W$$

Regional Bonus Decision Variables:

$$\sum_{i}^{Licenses\ in\ EA\ k} \left(\frac{x_i}{j}\right) \quad - \quad \alpha_{k,j} \quad \geq \quad 0$$

$$\sum_{k}^{EAs\ within\ Region\ r} \left(\frac{\alpha_{k,j}}{number\ of\ EAs\ in\ Region\ r}\right) \quad - \quad y_{r,j} \quad \geq \quad 0$$

National Bonus Decision Variables

$$\sum_{\boldsymbol{r}}^{Regions\ on\ map} \left(\frac{y_{r,j}}{number\ of\ Regions\ on\ map}\right) \quad - \quad z_{N,j} \quad \geq \quad \mathbf{0}$$

Currently (Provisionally) Winning Constraint

$$x_i \quad - \quad cw_i \quad \geq \quad 0$$

## *Appendix C: Illustration of Intra-round Bidding*

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Auctionomics/Power Auctions. *Supplement to Appendix C: Auctionomics/Power Auctions Option for Forward Auction*. 01 Feb. 2013.

To illustrate the use of intra-round bids, we provide the following example with six License Categories.

| License Category | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Target Supply | 9 | 9 | 9 | 9 | 9 | 9 |
| Start-of-round price | 40 | 20 | 40 | 60 | 40 | 20 |
| End-of-round price | 42 | 21 | 42 | 63 | 42 | 21 |

Suppose that the processed demands from the previous round are as follows:

| Processed demands (end of previous round) | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Bidder 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| Bidder 2 | 1 | 1 | 1 | 2 | 0 | 0 |
| All others | 8 | 9 | 8 | 10 | 12 | 10 |
| Aggregate Demand | 10 | 11 | 10 | 12 | 13 | 11 |

And suppose that the following intra-round bids are submitted in the current round:

| Bids | Price point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| Bidder 1 | 20% | –1 | 0 | 0 | 0 | 0 | 0 |
| Bidder 2 | 40% | 0 | –1 | 0 | 0 | 0 | 0 |
| Bidder 2 | 60% | –1 | 0 | +1 | 0 | 0 | 0 |
| Bidder 2 | 80% | 0 | 0 | 0 | –1 | +1 | +1 |

The first intra-round bid to be processed is Bidder 1's bid at the 20% price point, which is understood to be a proposed change in its demand from the 0% price point.

| Demands | Price point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| Bidder 1 | 0% | 1 | 1 | 1 | 0 | 1 | 1 |
| | 20% | −1 | 0 | 0 | 0 | 0 | 0 |
| Bidder 1 | 20% | 0 | 1 | 1 | 0 | 1 | 1 |
| Bidder 2 | 20% | 1 | 1 | 1 | 2 | 0 | 0 |
| All others | 20% | 8 | 9 | 8 | 10 | 12 | 10 |
| Aggregate | 20% | 9 | 11 | 10 | 12 | 13 | 11 |

Observe that Bidder 1's intra-round bid is feasible, as its application would result in Aggregate Demand that is at least the Target Supply of 9 for all License Categories. Next, Bidder 2's first bid, at the 40% price point, is processed as follows, noting that its processed demand from the previous round is carried forward and treated as its demand at the 0% price point of the current round. Meanwhile, Bidder 1's demand is unchanged from the 20% price point, and all other bidders' demands are unchanged from the previous round:

| Demands | Price point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| Bidder 2 | 0% | 1 | 1 | 1 | 2 | 0 | 0 |
| | 40% | 0 | −1 | 0 | 0 | 0 | 0 |
| Bidder 2 | 40% | 1 | 0 | 1 | 2 | 0 | 0 |
| Bidder 1 | 40% | 0 | 1 | 1 | 0 | 1 | 1 |
| All others | 40% | 8 | 9 | 8 | 10 | 12 | 10 |
| Aggregate | 40% | 9 | 10 | 10 | 12 | 13 | 11 |

Observe that Bidder 2's bid at the 40% price point is feasible, as it results in Aggregate Demand that is at least the Target Supply of 9 in all License Categories.

Next, Bidder 2's bid at the 60% price point is processed as follows, noting that Bidder 1's demand is still unchanged from the 20% price point, and all other bidders' demands are still unchanged from the previous round:

| Demands | Price point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| Bidder 2 | 40% | 1 | 0 | 1 | 2 | 0 | 0 |
| | 60% | −1 | 0 | +1 | 0 | 0 | 0 |
| Bidder 2 | 60% | 0 | 0 | 2 | 2 | 0 | 0 |
| Bidder 1 | 60% | 0 | 1 | 1 | 0 | 1 | 1 |
| All others | 60% | 8 | 9 | 8 | 10 | 12 | 10 |
| Aggregate | 60% | 8 | 10 | 11 | 12 | 13 | 11 |

Observe that Bidder 2's bid at the 60% price point is infeasible, as it would result in Aggregate Demand that is only 8—less than the Target Supply of 9—for License Category A. Consequently, no part of this bid is implemented, neither the decrease for License Category A nor the increase for License Category C. Instead, Bidder 2's bid at the 60% price point is placed in the queue.

Finally, Bidder 2's bid at the 80% price point is processed. The first processing step is adding its bid to its demand at the 40% price point, having rejected its bid at the 60% price point. Thus, the calculations are:

| Demands | Price point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| Bidder 2 | 40% | 1 | 0 | 1 | 2 | 0 | 0 |
| | 80% | 0 | 0 | 0 | −1 | +1 | +1 |
| Bidder 2 | 80% | 1 | 0 | 1 | 1 | 1 | 1 |
| Bidder 1 | 80% | 0 | 1 | 1 | 0 | 1 | 1 |
| All others | 80% | 8 | 9 | 8 | 10 | 12 | 10 |
| Aggregate | 80% | 9 | 10 | 10 | 11 | 14 | 12 |

After this (or any) intra-round bid is processed, another pass is made through bids in the queue to see if any can be feasibly applied. In this case, the one bid in the queue remains infeasible. There are no more bids to process, so bids in the queue are discarded and the round processing is over.

At the end of the round, for License Category A, Target Supply equals Demand. Since the last price point at which demand changed for this License Category was 20%, the Posted Price at the end of the round is 40.4, computed as 40+.2x(42-40). For all other License Categories, Demand exceeds Target Supply, and so the Posted Prices for those License Categories are equal to the end-of-round prices. The startof-round prices for the next round are therefore given as follows:

| License Category | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Target Supply | 9 | 9 | 9 | 9 | 9 | 9 |
| Next start-of-round price | 40.4 | 21 | 42 | 63 | 42 | 21 |

In summary, in this example, Bidder 1's bid, and two out of three of Bidder 2's bids, are applied; the remaining bid of Bidder 2 is not applied (and is ultimately discarded); and the price for License Category A rises by only 20% of the maximum increment for License Category A.

## *Appendix D: Sample Simulation Initial Conditions*

| | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 | Bidder 6 |
|---|---|---|---|---|---|---|
| **Bidder Type** | Regional | Regional | Regional | Regional | National | National |
| **Bidder Wealth** | 15.88632 | 16.49545 | 19.15201 | 19.65307 | 66.72035 | 77.32486 |
| $U_1=U_2=U_3$ | 3.101144 | 0 | 0 | 0 | 4.505474 | 3.584509 |
| $U_4=U_5=U_6$ | 3.853523 | 0 | 0 | 0 | 3.657613 | 3.14007 |
| $U_7=U_8=U_9$ | 3.696759 | 0 | 0 | 0 | 3.566189 | 4.220128 |
| $U_{10}=U_{11}=U_{12}$ | 4.465606 | 0 | 0 | 0 | 3.200879 | 3.693243 |
| $U_{13}=U_{14}=U_{15}$ | 0 | 4.348057 | 0 | 0 | 4.827679 | 3.85271 |
| $U_{16}=U_{17}=U_{18}$ | 0 | 4.295229 | 0 | 0 | 4.659775 | 4.820546 |
| $U_{19}=U_{20}=U_{21}$ | 0 | 4.257735 | 0 | 0 | 3.04585 | 4.676577 |
| $U_{22}=U_{23}=U_{24}$ | 0 | 4.12384 | 0 | 0 | 4.811158 | 4.34103 |
| $U_{25}=U_{26}=U_{27}$ | 0 | 0 | 4.49578 | 0 | 3.526719 | 3.252838 |
| $U_{28}=U_{29}=U_{30}$ | 0 | 0 | 3.94915 | 0 | 3.569675 | 3.378188 |
| $U_{31}=U_{32}=U_{33}$ | 0 | 0 | 3.873242 | 0 | 3.978383 | 4.480916 |
| $U_{34}=U_{35}=U_{36}$ | 0 | 0 | 3.014437 | 0 | 4.77692 | 4.237638 |
| $U_{37}=U_{38}=U_{39}$ | 0 | 0 | 0 | 3.561225 | 3.341211 | 3.269879 |
| $U_{40}=U_{41}=U_{42}$ | 0 | 0 | 0 | 4.338612 | 3.373532 | 4.796359 |
| $U_{43}=U_{44}=U_{45}$ | 0 | 0 | 0 | 3.374923 | 4.126072 | 3.977202 |
| $U_{46}=U_{47}=U_{48}$ | 0 | 0 | 0 | 3.232231 | 3.627398 | 3.527224 |
| $B_{1,1}$ | 4.497025 | 0 | 0 | 0 | 3.2666 | 3.436525 |
| $B_{1,2}$ | 6.733698 | 0 | 0 | 0 | 8.235233 | 8.503899 |
| $B_{1,3}$ | 1.541048 | 0 | 0 | 0 | 2.25707 | 2.434021 |
| $B_{2,1}$ | 0 | 4.258655 | 0 | 0 | 4.954904 | 4.197311 |
| $B_{2,2}$ | 0 | 8.049233 | 0 | 0 | 8.08296 | 7.983951 |
| $B_{2,3}$ | 0 | 2.232763 | 0 | 0 | 1.563959 | 1.947326 |
| $B_{3,1}$ | 0 | 0 | 3.665535 | 0 | 4.217847 | 3.822995 |
| $B_{3,2}$ | 0 | 0 | 9.815279 | 0 | 7.950792 | 9.153768 |
| $B_{3,3}$ | 0 | 0 | 1.475497 | 0 | 1.937182 | 1.593221 |
| $B_{4,1}$ | 0 | 0 | 0 | 3.977155 | 3.508695 | 4.294023 |
| $B_{4,2}$ | 0 | 0 | 0 | 7.59797 | 7.00498 | 8.551351 |
| $B_{4,3}$ | 0 | 0 | 0 | 1.289821 | 2.739496 | 2.606046 |
| $B_{N,1}$ | 0 | 0 | 0 | 0 | 11.13111 | 9.5903 |
| $B_{N,2}$ | 0 | 0 | 0 | 0 | 22.87537 | 20.77937 |
| $B_{N,3}$ | 0 | 0 | 0 | 0 | 7.440762 | 7.863533 |

## Appendix E: Sample Simulation Output

```
Starting Clock Auction IR with 16 Regions and 3 licenses per region
Initialized 6 Bidders
Starting Prices are as follows:
  Columns 1 through 11
    1.6245    1.7958    1.5514    1.9162    1.3159    1.9396    1.9860    1.1177    1.7090
1.7018    1.4446
  Columns 12 through 16
    1.3225    1.8583    1.9355    1.9555    1.1478
---------------Round 1 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    1.7869    1.9754    1.7066    2.1079    1.4475    2.1336    2.1846    1.2295    1.8799
1.8720    1.5891
  Columns 12 through 16
    1.4547    2.0441    2.1290    2.1511    1.2626
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
---------------Round 2 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    1.9656    2.1729    1.8773    2.3186    1.5922    2.3470    2.4031    1.3524    2.0678
2.0592    1.7480
  Columns 12 through 16
    1.6002    2.2485    2.3419    2.3662    1.3888
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
---------------Round 3 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.1622    2.3902    2.0650    2.5505    1.7515    2.5816    2.6434    1.4877    2.2746
2.2651    1.9228
  Columns 12 through 16
    1.7602    2.4733    2.5761    2.6028    1.5277
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
---------------Round 4 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.3784    2.6292    2.2715    2.8056    1.9266    2.8398    2.9078    1.6364    2.5021
2.4916    2.1151
  Columns 12 through 16
    1.9362    2.7207    2.8337    2.8631    1.6805
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
---------------Round 5 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.6162    2.8921    2.4986    3.0861    2.1193    3.1238    3.1985    1.8001    2.7523
2.7408    2.3266
  Columns 12 through 16
    2.1299    2.9927    3.1171    3.1494    1.8485
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
---------------Round 6 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.8778    3.1813    2.7485    3.3947    2.3312    3.4362    3.5184    1.9801    3.0275
3.0148    2.5592
  Columns 12 through 16
    2.3429    3.2920    3.4288    3.4643    2.0334
Demand exceeds supply for licenses in the following Economic Areas:
4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16,
---------------Round 7 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.8778    3.1813    2.7485    3.7342    2.5643    3.7798    3.8702    2.1781    3.3303
3.3163    2.8151
  Columns 12 through 16
```

```
    2.5771    3.2920    3.7717    3.4643    2.2367
Demand exceeds supply for licenses in the following Economic Areas:
3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16,
---------------Round 8 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.8778    3.1813    3.0233    3.7342    2.8207    4.1578    4.2573    2.3959    3.6633
3.6479    3.0967
  Columns 12 through 16
    2.8349    3.6212    4.1488    3.4643    2.4604
Demand exceeds supply for licenses in the following Economic Areas:
3, 5, 6, 7, 8, 9, 10, 11, 12, 16,
---------------Round 9 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    2.8778    3.1813    3.3257    3.7342    3.1028    4.5735    4.6830    2.6355    4.0296
4.0127    3.4063
  Columns 12 through 16
    3.1183    3.6212    4.1488    3.4643    2.7064
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 8, 13, 14, 15, 16,
---------------Round 10 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    3.1656    3.4995    3.6350    4.1076    3.1028    4.5735    4.6830    2.8990    4.0296
4.0127    3.4063
  Columns 12 through 16
    3.1183    3.9833    4.5637    3.8108    2.9770
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 4, 8, 13, 14, 15, 16,
---------------Round 11 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    3.3112    3.6605    3.6457    4.2966    3.1028    4.5735    4.6830    3.1889    4.0296
4.0127    3.4063
  Columns 12 through 16
    3.1183    4.1666    4.7736    3.9861    3.1140
Demand exceeds supply for licenses in the following Economic Areas:
6, 7, 8, 9, 10, 11, 12,
---------------Round 12 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    3.4600    3.8249    3.6566    4.4895    3.1028    5.0309    5.1513    3.5078    4.4326
4.4140    3.7470
  Columns 12 through 16
    3.4302    4.3537    4.9880    4.1651    3.2538
Demand exceeds supply for licenses in the following Economic Areas:
5, 6, 7, 8, 9, 10, 11, 12,
---------------Round 13 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    3.4822    3.8494    3.6582    4.5184    3.4131    5.3076    5.4346    3.7007    4.6764
4.6568    3.9530
  Columns 12 through 16
    3.6188    4.3691    5.0057    4.1798    3.2653
Demand exceeds supply for licenses in the following Economic Areas:
1, 2, 3, 4, 5,
---------------Round 14 has ended-------------------
Prices are as follows:
  Columns 1 through 11
    3.8304    4.2344    4.0241    4.9702    3.7544    5.5340    5.6664    3.8586    4.8759
4.8554    4.1217
  Columns 12 through 16
    3.7732    4.3817    5.0201    4.1918    3.2747
Demand exceeds supply for licenses in the following Economic Areas:
---
Auction has ended... exporting results.
Total Bidder Utility:
227.187
Total Revenue:
233.722
```